

## Chapter 1. Precalculus review

### 1.7 Composite functions

#### Arithmetic operations with functions

$$\begin{array}{ll}
 f + g : & (f + g)(x) = f(x) + g(x), & \text{dom}(f + g) = \text{dom}(f) \cap \text{dom}(g), \\
 fg : & (fg)(x) = f(x)g(x), & \text{dom}(fg) = \text{dom}(f) \cap \text{dom}(g), \\
 \frac{f}{g} : & \left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, & \text{dom}(f/g) = \text{dom}(f) \cap \{x \in \text{dom}(g), g(x) \neq 0\}.
 \end{array}$$

#### Composition of functions

$$f \circ g : \quad (f \circ g)(x) = f(g(x)), \quad \text{dom}(f \circ g) = \{x \in \text{dom}(g), g(x) \in \text{dom}(f)\}.$$

NOTE: Transforming functions be carefull not to change its domain.

Compare domains of

$$\begin{array}{ll}
 (\sqrt{x})^2 & \text{and} & x, \\
 \frac{\sqrt{x-1}}{\sqrt{x+1}} & \text{and} & \sqrt{\frac{x-1}{x+1}}.
 \end{array}$$

#### Inverse function

Function  $g$  is called inverse to  $f$  if  $x = g(y)$  is the solution of equation  $f(x) = y$ ; here  $y \in \text{dom}(g)$  iff equation  $f(x) = y$  has exactly one solution.

NOTE Definition  $g(f(x)) = x$  means that equation  $f(x) = y$  should have no more than one solution for any  $y$  and that  $g(y)$  is this solution if exists but it does not say that  $y \notin \text{dom}(g)$  if this solution does not exist.

Another definition  $f(g(y)) = y$  means that  $g(y)$  provides a solution to equation  $f(x) = y$  but does not claim that there are no other solutions.

:

How one should restrict domain of  $f$  in order to have an inverse function defined everywhere of range( $f$ ) where

$$\begin{array}{l}
 f(x) = x^2; \\
 f(x) = (x^2 - 1)^2; \\
 f(x) = \sin x; \\
 f(x) = \cos x;
 \end{array}$$

(there could be different correct answers).