

Chapter 1. Precalculus review

1.6 (Very) Elementary Functions

(continued)

Trigonometric functions

Questions to be able to answer:

- What is π and what is the length of the the circumference? (avoid vicious circle)
- How to measure angles and arcs?
- What are $\cos x$, $\sin x$, $\tan x$, $\cot x$, $\sec x$, $\csc x$?
- What are domains and ranges of these functions?
- Which of them are even? odd?
- All of these functions are 2π -periodic but some of them are π -periodic. Which? What happens with other functions if we add π to its argument?

Appendix: Useful formulae to remember:

$$(1) \quad \cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

$$(2) \quad \sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$

$$(3) \quad \tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$$

one can find similar formulae for cot but there is no point to remember;

$$(4) \quad \cos x = \pm \sqrt{1 - \sin^2 x} = \pm \frac{1}{\sqrt{1 + \tan^2 x}}$$

$$(5) \quad \sin x = \pm \sqrt{1 - \cos^2 x} = \pm \sqrt{1 - \sin^2 x}$$

the following three are due to the first suite:

$$(6) \quad \cos x \cos y = \frac{1}{2}(\cos(x + y) + \cos(x - y)),$$

$$(7) \quad \sin x \sin y = \frac{1}{2}(\cos(x - y) - \cos(x + y)),$$

$$(8) \quad \cos x \sin y = \frac{1}{2}(\sin(x + y) + \sin(x - y)),$$

and they are obviously equivalent to the following three:

$$(9) \quad \cos x + \cos y = 2 \cos \frac{x + y}{2} \cos \frac{x - y}{2}$$

$$(10) \quad -\cos x + \cos y = 2 \sin \frac{x + y}{2} \sin \frac{x - y}{2}$$

$$(11) \quad \sin x \pm \sin y = 2 \sin \frac{x \pm y}{2} \cos \frac{x \mp y}{2}$$

the following three are due to the first suite:

$$(12) \quad \cos(2x) = \cos^2 x - \sin^2 x = 2 \cos^2 x - 1 = 1 - 2 \sin^2 x$$

$$(13) \quad \sin(2x) = 2 \sin x \cos x$$

$$(14) \quad \tan(2x) = \frac{2 \tan x}{1 - \tan^2 x}$$

and they imply last three:

$$(15) \quad \cos \frac{x}{2} = \pm \sqrt{\frac{1 + \cos x}{2}}$$

$$(16) \quad \sin \frac{x}{2} = \pm \sqrt{\frac{1 - \cos x}{2}}$$

$$(17) \quad \tan \frac{x}{2} = \sqrt{\frac{1 - \cos x}{1 + \cos x}}$$

Appendix: Greeks:

dimmed letters coincide with Latin and are not used in math; on the other hand, cunning mathematicians consider different variants of some small letter as different letters; similarly l and ℓ are different too

A	α	alpha
B	β	beta
Γ	γ	gamma
Δ	δ	delta
E	ϵ, ε	epsilon
Z	ζ	zeta
H	η	eta
Θ	θ, ϑ	theta
I	ι	iota
K	κ, \varkappa	kappa
Λ	λ	lambda
M	μ	mu
N	ν	nu
Ξ	ξ	xi
O	o	omicron
Π	π, ϖ	pi
P	ρ, ϱ	rho
Σ	σ, ς	sigma
T	τ	tau
Υ	υ	upsilon
Φ	ϕ, φ	phi
X	χ	chi
Ψ	ψ	psi
Ω	ω	omega

PS. If someone does not like Greeks letters, tell me: I can allways use German fraktur instead:

$\mathfrak{A}, \mathfrak{B}, \mathfrak{C}, \mathfrak{D}, \mathfrak{E}, \mathfrak{F}, \mathfrak{G}, \mathfrak{H}, \mathfrak{I}, \mathfrak{J}, \mathfrak{K}, \mathfrak{L}, \mathfrak{M}, \mathfrak{N}, \mathfrak{O}, \mathfrak{P}, \mathfrak{Q}, \mathfrak{R}, \mathfrak{S}, \mathfrak{T}, \mathfrak{U}, \mathfrak{V}, \mathfrak{W}, \mathfrak{X}, \mathfrak{Y}, \mathfrak{Z}$
 $\mathfrak{a}, \mathfrak{b}, \mathfrak{c}, \mathfrak{d}, \mathfrak{e}, \mathfrak{f}, \mathfrak{g}, \mathfrak{h}, \mathfrak{i}, \mathfrak{j}, \mathfrak{k}, \mathfrak{l}, \mathfrak{m}, \mathfrak{n}, \mathfrak{o}, \mathfrak{p}, \mathfrak{q}, \mathfrak{r}, \mathfrak{s}, \mathfrak{t}, \mathfrak{u}, \mathfrak{v}, \mathfrak{w}, \mathfrak{x}, \mathfrak{y}, \mathfrak{z}$