# Chapter 7

## 7.8 Inverse Hyperbolic Functions

### Defintion

Obviously equations $\sinh x = t$, $\tanh x = t$ and $\cosh x = t$ have respectively exactly one solution, no more than one solution, no more than two solutions.

**Definition 1.** We introduce uniquely the following trigono,etric functions:

- $\sinh^{(-1)} : (-\infty, +\infty) \to (-\infty, +\infty) \ni y = \sinh^{(-1)}(x) = \log(x+\sqrt{x^2+1})$ iff $\sinh x = y$;
- $\cosh^{(-1)} : [1, \infty] \to [0, +\infty) \ni \cosh^{(-1)}(x) = \log(x + \sqrt{x^2-1})$ iff $\cosh x = y$, $x \geq 1$;
- $\tanh^{(-1)} : (-1, 1) \to (-\infty, +\infty) \ni y = \tanh^{(-1)}(x) = \frac{1}{2}\log\frac{1-x}{1+x}$ iff $\tanh x = y$;
- $\coth^{(-1)}) : (-\infty, 1) \cup (1, +\infty) \to (-\infty, 0) \cup (0, +\infty) \ni y = \coth^{(-1)}(x) = \frac{1}{2}\log\frac{x-1}{x+1}$ iff $\coth x = y$;

### Differentiation

**Theorem 2.** *Inverse hyperbolic functions have derivatives*

$$(1) \qquad \left(\sinh^{(-1)}(x)\right)' = \frac{1}{\sqrt{x^2+1}},$$

$$(2) \qquad \left(\cosh^{(-1)}(x)\right)' = \frac{1}{\sqrt{x^2-1}} \qquad x > 1,$$

$$(3) \qquad \left(\sinh^{(-1)}(\frac{1}{x})\right)' = -\frac{1}{|x|\sqrt{x^2+1}},$$

$$(4) \qquad \left(\cosh^{(-1)}(\frac{1}{x})\right)' = -\frac{1}{x\sqrt{1-x^2}} \qquad 0 < x < 1,$$

$$(5) \qquad \left(\tanh^{(-1)}(x)\right)' = \frac{1}{1-x^2} \qquad |x| < 1,$$

$$(6) \qquad \left(\coth^{(-1)}(x)\right)' = \frac{1}{1-x^2} \qquad |x| > 1.$$

*Proof.* Follows from table derivatives. □

## Graphs of Hyperbolic Functions

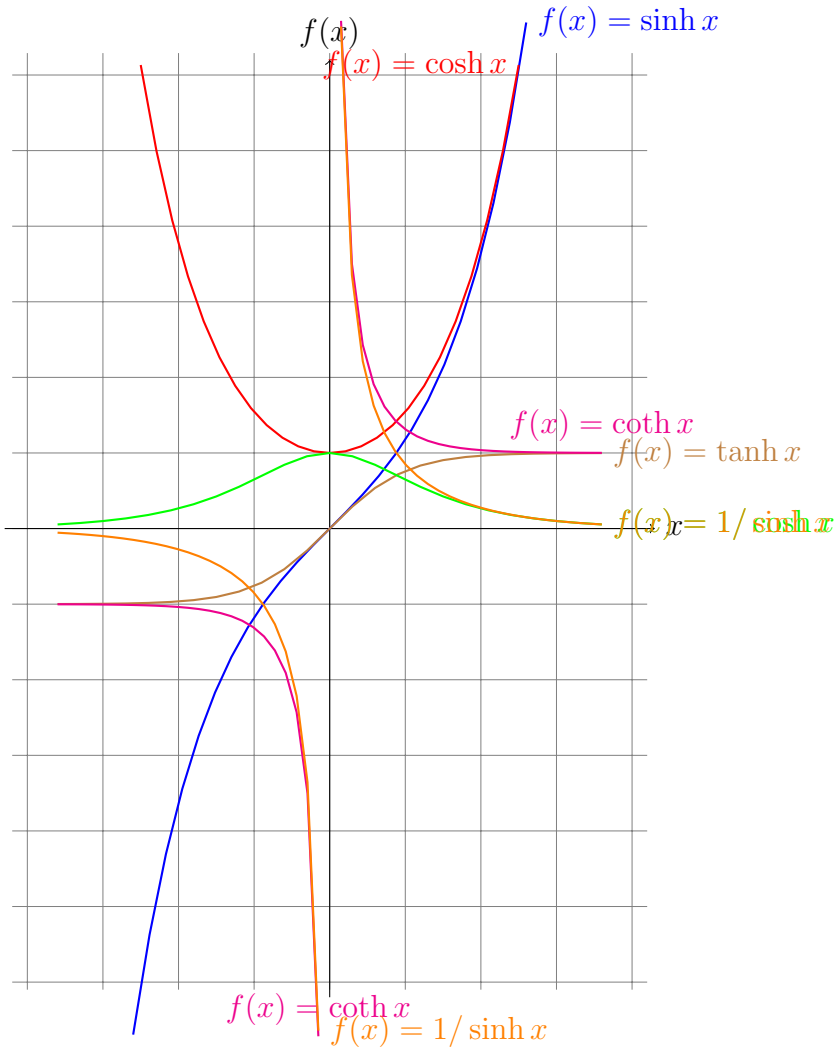


Figure 1: Trigonometric functions