Chapter 1. Precalculus review

1.6 (Very) Elementary Functions

Polynomials

Questions to be able to answer:

- What is a polynomial P_n? Degree of polynomial?
- What is $\operatorname{dom} P_n$? $\operatorname{range} P_n$?
- How graph of P_n behaves at $\pm\infty$?
- Division of polynomials with remainder
- Do you know that P_n is divisible by $(x-r)$ iff r is the root of P_n?
- Simple roots, multiple (double, triple, ...) roots.
- How behaves graph of polynomials near roots?
- How many (real) roots can have polynomial?
- How many maxima/minima can have polynomial?

Rational Functions

Rational function is

$$f(x) = \frac{P_n(x)}{Q_m(x)} \tag{1}$$

- Is polynomial a rational function?
- When f is even/odd function?
- What is $\operatorname{dom} f$?
- We can reduce polynomials P_n and Q_m by the common factor; it can happen that some roots of Q_m will disappear. To be completely rigorous we must not include them into domain (or to make a remark that we extended naturally domain of f) but we will. From now on (1) is *irreducible*.
- How behaves graph of f near roots of P_n?
- How behaves graph of f near roots of Q_m?
- How behaves graph of f at $\pm\infty$?

- Which operations preserve the class of rational functions:
  - Is the sum of rational functions rational function?
  - Is the product of rational functions rational function?
  - Is the derivative of rational functions rational function?
  - Is the primitive (integral) of rational functions rational function?

Trig functions will be c onsidered next time