Optimization & Indeterminate forms and L'Hôpital's Rule

UNIVERSITY OF TORONTO

November 27th, 2018

Announcements

1. For next class: watch the videos 6.8 through 6.10.

2. Ivan's office hours of this afternoon are cancelled. Check the course website for up-to-date info about the office hours.

From last week (1)

Theorem

Let $a < b$. Let f be a function differentiable on (a, b).

- IF $\forall x \in (a, b)$, $f'(x) > 0$,
- THEN f is increasing on (a, b).

(Bad) proof:

- From the MVT, there exists $c \in (a, b)$ such that $f'(c) = \frac{f(b)-f(a)}{b-a}$.
- We know that $f'(c) > 0$ and that $b - a > 0$.
- So $f(b) - f(a) = f'(c)(b - a) > 0$ and $f(b) > f(a)$.
- Hence f is increasing.

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What's wrong with this proof?

From last week (2)

Write a correct proof!

Theorem

Let $a < b$. Let f be a function differentiable on (a, b).

- IF $\forall x \in (a, b)$, $f'(x) > 0$,
- THEN f is increasing on (a, b).

1. Write formally what we want to show.
2. Write the structure of the proof.
3. Rough work!
4. Write a correct proof!

Distance

Find the point on the parabola $y^2 = 2x$ that is the closest to the point $(1, 4)$.

The classic farmer problem

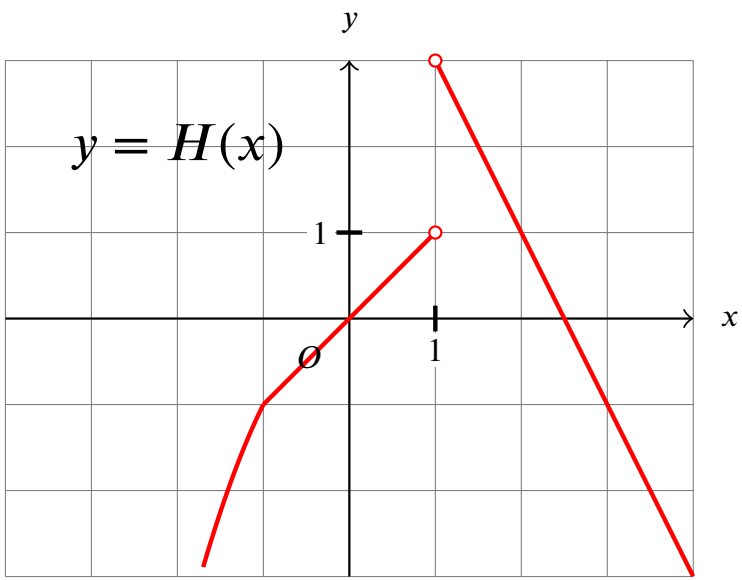
A farmer has $300m$ of fencing and wants to fence off a rectangular field and add an extra fence that divides the rectangular area in two equal parts down the middle. What is the largest area that the field can have?

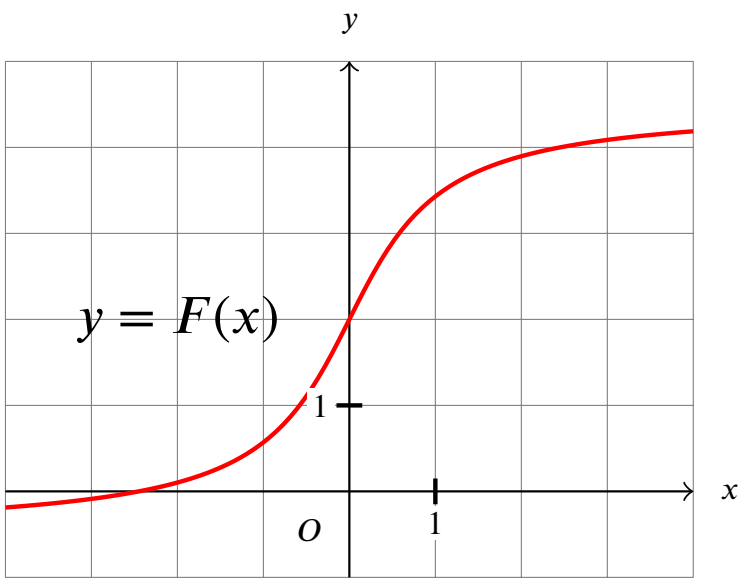
Limits from graphs

Compute:

1. $\lim_{x \to 0} \dfrac{H(x)}{H(2+3x)-1}$

2. $\lim_{x \to 2} \dfrac{F^{-1}(x)}{x-2}$





Computations

Compute:

1. $\lim_{x\to 2} \dfrac{x^2+2x-6}{x^2+3x-10}$
2. $\lim_{x\to 0} \dfrac{e^{2x^2}-\cos x}{x\sin x}$
3. $\lim_{x\to\infty} \dfrac{x^2}{e^x}$
4. $\lim_{x\to\infty} \dfrac{e^x+e^{-x}}{e^x-e^{-x}}$
5. $\lim_{x\to\infty} (\sin x)\left(e^{1/x}-1\right)$
6. $\lim_{x\to\infty} x\sin\dfrac{2}{x}$
7. $\lim_{x\to\infty} x\cos\dfrac{2}{x}$
8. $\lim_{x\to 1}\left[(\ln x)\tan\dfrac{\pi x}{2}\right]$