The Multivariable Alexander Polynomial:

- multivariable version of the classical Alexander polynomial
- invariant for long, regular, virtual knots and links
- vMVA (L) ∈ ℤ[t, t⁻¹]

GOAL: Study its generalizations to tangles.

First generalization (J. Archibald '06)

ψ Tₙ regular v-tangles with ends labelled by \(|X^{in}| = |X^{out}| = n\)

AHD(\(X^{in}, X^{out}\)) = \(\Lambda^n(\mathbf{0}) \otimes \Lambda^n(\mathbf{1}) \otimes \Lambda^n(\mathbf{0}) \otimes \Lambda^n(\mathbf{1})\)

Tangle invariant, which is a circuit algebra morphism:

tMVA : (\(\psi T_n\), gluing) → (AHD(\(X^{in}, X^{out}\)), interior mult.)

Oriented Circuit Algebra

An OCA is a collection \(\mathcal{V}\) of objects indexed by pairs \((n, m) \in \mathbb{N}^2\)
and morphisms \(\mathcal{F}\) indexed by circuit diagrams:

The Alexander matrix \(M(D_T)\) is built via the rules:

\[
M(D_T) = \begin{pmatrix}
X^{in} & X^{out} \\
S_1 & S_2 \\
\end{pmatrix}
\]

where:

- \(X^{in}\) label the internal, incoming, and outgoing arcs
- \(\{t_i\}\) associated to the strands

The Alexander matrix \(M(D_T)\) is built via the rules:

Definition of tMVA

For any regular, virtual tangle \(T\) with \(m\) strands, \((n, m)\):

tMVA\(T\) = \(\prod_{k=1}^m t^{\mu(k)/2} w \otimes \sum_{\lambda \in \mathcal{L}} M(D_T)^{\mu(k) \otimes \lambda} \otimes \ldots \otimes s_{n_k}\)

where:

- \(w \in \Lambda^n(\mathbf{0})\)
- \(M(D_T)^{\mu(k) \otimes \lambda} \) minor with columns \(I\) and \(s_{n_1}, \ldots, s_{n_k} \subseteq X^{out} \cup X^{in}\)

Properties of tMVA

- It is a circuit algebra morphism.
- Satisfies “Overcrossings Commute”, so is a \(w\)-tangle invariant.
- For \(u\)-tangles, can get R1 invariance: tMVA’ = \(\prod t^{\mu(k)/2} w\) tMVA
- Can recover vMVA:

\[
tMVA\left(\begin{pmatrix} a & b \end{pmatrix}\right) = vMVA\left(\begin{pmatrix} a & b \end{pmatrix}\right)\left(b \otimes a \otimes b \otimes a\right)
\]

- Gives easy verification of many local vMVA relations

Future directions:

OCA vs. MM, extend \(t_{\lambda}\) to \(\zeta\), \(\lambda = 0\), categorification?