

Mathematical Introduction to Game Theory

Assignment 6, due November 14

Problem 1 of 5. Let us consider a general sum $n \times m$ game.

a): Prove that for a fixed $\mathbf{p} \in \Delta^n$ the set of points

$$\{\mathbf{q} \in \Delta^m : (\mathbf{p}, \mathbf{q}) \text{ is a Nash equilibrium.}\}$$

is convex.

b): Show that the set

$$\{(\mathbf{p}, \mathbf{q}) \in \Delta^n \times \Delta^m : (\mathbf{p}, \mathbf{q}) \text{ is a Nash equilibrium.}\}$$

does not have to be convex.

Problem 2 of 5. Consider the 3×3 two-person non-zero-sum game with payoff matrix:

$$\begin{pmatrix} (1, 0) & (-1, 0) & (0, 1) \\ (0, 0) & (1, 1) & (-1, -3) \\ (0, 2) & (0, -2) & (5, 4) \end{pmatrix}$$

Find a Nash equilibrium.

Problem 3 of 5. Consider the 3×3 two-person non-zero-sum game with payoff matrix:

$$\begin{pmatrix} (3, 2) & (3, 0) & (2, 2) \\ (1, 0) & (2, 3) & (0, 3) \\ (0, 2) & (0, 0) & (3, 2) \end{pmatrix}$$

Find all the Nash equilibria (pure and mixed) of the game.

Problem 4 of 5. Players I and II are given a card at random. Each card is a *Winning* card with probability $1/3$, and a *Losing* card with probability $2/3$. After looking at their cards, without seeing the card of the other player, each player tries to guess the card of the other player. If players have the same cards (both Winning or both Losing), each player who made a correct guess is paid \$2 and each player who made an incorrect guess loses \$1. Otherwise, each player who made a correct guess is paid \$1 and each player who made an incorrect guess loses \$1.

- (1) Draw the Kuhn tree.
- (2) Find the equivalent strategic form.
- (3) Find the safety levels.
- (4) Find all the Nash Equilibria.

Problem 5 of 5. Consider the following model of duopoly.

The market has capacity A for a certain good. The production cost of each unit equal to C_1 for the company I and $C_2 < C_1$ for the company II. The price of the product is equal to $A - Q$, where Q is the total number of the units produced. Company I makes a decision about the number of units it will produce and informs Company II about its decision. The company II then makes the decision about the number of units it will produce.

Analyze the model by finding all the Nash equilibria, and comparing the consumer prices and profits with the case of the monopoly of the company I.