

MAT406H5F. Assignment 6, due November 12

Problem 1 of 5

Let us consider a general sum game $m \times n$ game.

a) Prove that for a fixed $\mathbf{p} \in \Delta^m$ the set of points

$$\{\mathbf{q} \in \Delta^n : (\mathbf{p}, \mathbf{q}) \text{ is a Nash equilibrium.}\}$$

is convex.

b) Show that the set

$$\{(\mathbf{p}, \mathbf{q}) \in \Delta^m \times \Delta^n : (\mathbf{p}, \mathbf{q}) \text{ is a Nash equilibrium.}\}$$

does not have to be convex.

Problem 2 of 5

Consider the 3×3 two-person non-zero-sum game with payoff matrix:

$$\begin{pmatrix} (3, 2) & (3, 0) & (2, 2) \\ (1, 0) & (3, 3) & (0, 3) \\ (0, 2) & (0, 0) & (3, 2) \end{pmatrix}$$

Find all the Nash equilibria (pure and mixed) of the game.

Hint: There is no strict domination here, but what can you say about a Nash equilibrium which gives nonzero weight to the second strategy of the first player?

Problem 3 of 5

A war of attrition. Two players are involved in a dispute over an object. The value of the object to player i is $v_i > 0$. Time is modeled as a continuous variable that starts at 0 and runs indefinitely. Each player chooses when to concede the object to the other player; if the first player to concede does so at time t , the other player obtains the object at that time. If both players concede simultaneously, the object is split equally between them, player i receiving a payoff of $v_i/2$. Time is valuable: until the first concession each player loses one unit of payoff per unit of time. Formulate this situation as a strategic game and show that in all Nash equilibria one of the players concedes immediately.

Problem 4 of 5

Consider the following model of duopoly.

The market has capacity A for a certain good. The production cost of each unit equal to C_1 for the company I and $C_2 < C_1$ for the company II. The price of the product is equal to $A - Q$, where Q is the total number of the units produced. Company I makes a decision about the number of units it will produce and informs Company II about its decision. The company II then makes the decision about the number of units it will produce.

Analyze the model by finding all Nash equilibria, and comparing the consumer prices and profits with the case of the monopoly of the company I.

Problem 5 of 5

Considering the game in Problem 3 as an NTU cooperative game, describe the feasible payoff set. Describe all Pareto optimal payoff vectors.