Complex Analysis

Assignment 5, due November 9

Problem 1 of 5. Problem 4, page 108 of *Ahlfors*.

Problem 2 of 5. Let f be a continuous function in a disk $B(z_0, r)$, let γ^δ be the circle $\{|z - z_0| = \delta\}$ oriented counterclockwise. Prove that

$$\lim_{\delta \to 0} \oint_{\gamma^\delta} \frac{f(z)}{z - z_0}\, dz = 2\pi i f(z_0).$$

Problem 3 of 5. Let γ be a curve from 0 to 1 not passing through i or $-i$. Prove that

$$\oint_\gamma \frac{dz}{1 + z^2} = \frac{\pi}{4} + k\pi, \quad k \in \mathbb{Z}.$$

Problem 4 of 5. Assume that f is a continuous function on the closed disk $\overline{B(z_0, r)}$ which is analytic on the open disk $B(z_0, r)$. Prove that

$$\oint_{C_r} f(\xi)\, d\xi = 0,$$

where C_r is the positively oriented circle of radius r centered at z_0.
Be careful: you cannot assume that f is analytic in the closed disk $\overline{B(z_0, r)}$.

Problem 5 of 5. Let $\Gamma : [a,\, b] \times [0,\, 1] \mapsto \mathbb{C}$ be a continuous function with $\Gamma(a, s) = \Gamma(b, s)$ for all $0 \le s \le 1$. Let $\gamma_1(t) = \Gamma(t, 0),\ \gamma_2(t) = \Gamma(t, 1) : [a,\, b] \mapsto \mathbb{C}$ be two closed curves. Let $z_0 \notin \Gamma\Big([a,\, b] \times [0,\, 1]\Big)$. Show that

$$n(\gamma_1, z_0) = n(\gamma_2, z_0).$$

Remark: Γ is called a *homotopy* between γ_1 and γ_2.