Complex Analysis

Assignment 3, due October 4

Problem 1 of 5. Let

$$p(z) = a_d z^d + a_{d-1} z^{d-1} + \cdots + a_0$$

be a polynomial of degree d. Use Gauss' Theorem to prove that for any $w \in \mathbb{C}$ the equation $p(z) = w$ has a solution with

$$|z| \geq \left| \frac{a_{d-1}}{d a_d} \right|.$$

Problem 2 of 5. Problem 4, page 33 of *Ahlfors*.

Problem 3 of 5. Problem 6, page 33 of *Ahlfors*.

Problem 4 of 5. Let $\sum_{n=0}^{\infty} a_n z^n$ and $\sum_{n=0}^{\infty} b_n z^n$ be two power series with the radii of convergence R_1 and R_2 correspondingly. Prove that

(1) The radius of convergence R of the series $\sum_{n=0}^{\infty} (a_n + b_n) z^n$ satisfies $R \geq \min(R_1, R_2)$ and that equality holds if $R_1 \neq R_2$.

(2) The radius of convergence R of the series $\sum_{n=0}^{\infty} (a_n b_n) z^n$ satisfies $R \geq R_1 R_2$.

Problem 5 of 5. Determine the radius of convergence of each of the following series:

(1) $\sum_{n=1}^{\infty} \left(\frac{7n^4 + 2n^3}{3n^4 + 2n} \right) z^n$

(2) $\sum_{n=1}^{\infty} (n^2 + a^n) z^n$, $a \in \mathbb{C}$

(3) $\sum_{n=1}^{\infty} (\sin n) z^n$