Complex Analysis

Assignment 2, due September 28

Problem 1 of 5. Let g be a continuous function on a disk $B(z_0, \delta)$, and f be a complex-differentiable function at the point $g(z_0)$, which satisfies the relation $f(g(z)) = z$ for all $z \in B(z_0, \delta)$. Assume that $f'(g(z_0)) \neq 0$. Show that g is complex-differentiable at z_0 and

$$g'(z_0) = \frac{1}{f'(g(z_0))}.$$

Problem 2 of 5. Let f be defined in some disk $B(z_0, \delta)$ and real-differentiable at z_0. Assume that

$$\lim_{z \to z_0} \left| \frac{f(z) - f(z_0)}{z - z_0} \right|$$

exists. Show that either f or $\overline{f}$ is complex-differentiable at z_0.

Problem 3 of 5. Problem 4, page 28 of *Ahlfors*.

Problem 4 of 5. Problem 5, page 28 of *Ahlfors*.

Problem 5 of 5. Problem 7, page 28 of *Ahlfors*.