Problem 1 of 5. Use the cotangent trick to compute
\[ \sum_{n=1}^{\infty} \frac{1}{n^6 + n^2}. \]
Be careful at \( z = 0 \! \).

Problem 2 of 5.
(1) Let \( u, v \) be two subharmonic functions in \( \Omega \). Show that \( \max(u, v) \) is also subharmonic in \( \Omega \).
(2) Show that if a non-constant \( u \in \text{Harm}(\Omega) \), then \( u \) can not have local maxima or minima inside \( \Omega \).
(3) Give an example of non-constant bounded subharmonic function \( v \) in the unit disk (i.e. a bounded subharmonic function such that for some \( z_1, z_2 \in \mathbb{D}, v(z_1) \neq v(z_2) \)) which has a local maximum in \( \mathbb{D} \).

Problem 3 of 5. Problem 2, page 166 of Ahlfors.


Problem 5 of 5. Let \( f(z) = \sum_{n=-\infty}^{\infty} a_n z^n \) be an analytic function in \( \mathbb{C} \setminus \{0\} \). Assume that
\[ \forall N \in \mathbb{N} \exists m > N, n > N : a_n \neq 0, a_{-m} \neq 0. \]
Let \( M(r) := \max_{|z|=r} |f(z)| \). Show that for any \( k \in \mathbb{N} \)
\[ \lim_{r \to 0} \frac{1}{r^k M(r)} \quad \text{and} \quad \lim_{r \to \infty} \frac{r^k}{M(r)} = 0. \]