Complex Analysis

Assignment 8, due April 1

Problem 1 of 5. Use the cotangent trick to compute

$$\sum_{n=1}^{\infty} \frac{1}{n^6 + n^2}.$$

Be careful at $z = 0$!

Problem 2 of 5.

(1) Let u, v be two subharmonic functions in Ω. Show that $\max(u, v)$ is also subharmonic in Ω.

(2) Show that if a non-constant $u \in Harm(\Omega)$, then u can not have local maxima or minima inside Ω.

(3) Give an example of non-constant bounded subharmonic function v in the unit disk (i.e. a bounded subharmonic function such that for some z_1, $z_2 \in \mathbb{D}$, $v(z_1) \neq v(z_2)$) which ahs a local maximum in $\mathbb{D}$.

Problem 3 of 5. Problem 2, page 166 of *Ahlfors*.

Problem 4 of 5. Problems 1-2, page 171 of *Ahlfors*.

Problem 5 of 5. Let $f(z) = \sum_{n=-\infty}^{\infty} a_n z^n$ be an analytic function in $\mathbb{C} \setminus \{0\}$. Assume that

$$\forall N \in \mathbb{N}\, \exists m > N, n > N \,:\, a_n \neq 0, a_{-m} \neq 0.$$

Let $M(r) := \max_{|z|=r} |f(z)|$. Show that for any $k \in \mathbb{N}$

$$\lim_{r \to 0} \frac{1}{r^k M(r)} = \lim_{r \to \infty} \frac{r^k}{M(r)} = 0.$$