Complex Analysis
Assignment 3, due February 4

Problem 1 of 5. Determine the radius of convergence of each of the following series:

(1) $\sum_{n=1}^{\infty} \left(\frac{7n^4 + 2n^3}{3n^4 + 2n} \right) z^n$

(2) $\sum_{n=1}^{\infty} \left(n^2 + a^n \right) z^n$, $a \in \mathbb{C}$

(3) $\sum_{n=1}^{\infty} \left(\sin n \right) z^n$

Problem 2 of 5. Let $\sum_{n=0}^{\infty} a_n z^n$ and $\sum_{n=0}^{\infty} b_n z^n$ be two power series with the radii of convergence R_1 and R_2 correspondingly. Prove that

(1) The radius of convergence R of the series $\sum_{n=0}^{\infty} (a_n + b_n) z^n$ satisfies $R \geq \min(R_1, R_2)$ and that equality holds if $R_1 \neq R_2$.

(2) The radius of convergence R of the series $\sum_{n=0}^{\infty} (a_n b_n) z^n$ satisfies $R \geq R_1 R_2$.

Problem 3 of 5. Problem 8, page 41 of *Ahlfors*.

Problem 4 of 5. Problem 4, page 44 of *Ahlfors*.

Problem 5 of 5. Problem 6, page 47 of *Ahlfors*.