Complex Analysis

Assignment 2, due January 28

Problem 1 of 5. Let g be a continuous function on a disk $B(z_0, r)$, and f be a complex-differentiable function at the point $g(z_0)$, which satisfies the relation $f(g(z)) = z$ for all $z \in B(z_0, r)$. Assume that $f'(g(z_0)) \neq 0$. Show that g is complex-differentiable at z_0 and

$$g'(z_0) = \frac{1}{f'(g(z_0))}.$$

Problem 2 of 5. Let f be defined in some disk $B(z_0, r)$ and real-differentiable at z_0. Assume that

$$\lim_{z \to z_0} \left| \frac{f(z) - f(z_0)}{z - z_0} \right|$$

exists. Show that either f or $\overline{f}$ is complex-differentiable at z_0.

Problem 3 of 5. Problem 7, page 28 of *Ahlfors*.

Problem 4 of 5. Problems 2 and 3 on page 32 of *Ahlfors*.

Problem 5 of 5. Let $\sum_{n=1}^{\infty} z_n$ be a convergent series of complex numbers.

(1) Let $\alpha < \pi/2$. Show that if for all n, $|\operatorname{Arg} z_n| < \alpha$, then the series $\sum_{n=1}^{\infty} z_n$ converges absolutely.

(2) Does the conclusion of the previous part hold if $\alpha = \pi/2$?

(3) Assume that for all n, $\Re z_n \geq 0$ and that the series $\sum_{n=1}^{\infty} z_n^2$ also converges. Show that the series $\sum_{n=1}^{\infty} z_n^2$ converges absolutely.