

Final Exam, MAT334

Name: _____

December 13

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Problem 1

Solve the equation

$$z^3 - z^2 + iz - i = 0.$$

Problem 2

Find the function $u(x, y)$ that is harmonic at the infinite strip

$$\{z : 0 < \text{Im}z < 1\},$$

and take the value 0 at the bottom edge and the value 2 at the top edge.

Problem 3

Find all the values of

$$(1 + i)^{1-i}.$$

Write the answer in polar form.

Problem 4

Compute

$$\oint_{\gamma} 2\bar{z} - 2 dz,$$

where γ is the straight line segment joining 1 and i .

Problem 5

Using the partial fraction expansion, compute

$$\oint_{\gamma} \frac{1}{(z-1)(z+1)} dz,$$

where γ is the circle $|z| = 2$ traversed once in clockwise direction.

Note: you will get no credit for this problem if you use residues for the computation.

Problem 6

Find all functions f analytic in the unit disk that satisfy $f(0) = 3 + 4i$, $|f(z)| \leq 5$.

Problem 7

Find the Taylor series, and state the convergence properties for the function $f(z) = \sin z$ around $z_0 = \pi$.

Problem 8

Find the Laurent series for the function $f(z) = \exp\left(\frac{1}{z} - 1\right)$ in the domain $1 < |z| < 2$.

Problem 9

Suppose that $f(z)$ has a pole at 0. What type of singularity does the function $e^{-f(z)}$ has at 0? Justify your statement.

Problem 10

Compute

$$\int_0^\pi \frac{d\theta}{10 + 8 \cos \theta}$$