FACULTY OF ARTS AND SCIENCE
University of Toronto

FINAL EXAMINATIONS, APRIL/MAY 2007

MAT 133Y1Y

Calculus and Linear Algebra for Commerce

PART A. MULTIPLE CHOICE

1. *[3 marks]*

The system of equations

$$\begin{array}{rcrcrcrcrcl} x & + & y & + & z & + & u & + & v & = & 1 \\ & & y & + & z & + & 2u & & & = & 2 \\ & & & & z & + & u & + & 2v & = & 3 \end{array}$$

has

Ⓐ no solutions

Ⓑ a unique solution

Ⓒ infinitely many solutions with one parameter

Ⓓ infinitely many solutions with two parameters

Ⓔ infinitely many solutions with three parameters

2. *[3 marks]*

Let $h(x) = \begin{cases} \dfrac{x^2-1}{x-1} & \text{for } x \neq 1 \\ a-2 & \text{for } x = 1 \end{cases}$

Then $h(x)$ is continuous everywhere for a equal to

Ⓐ 4

Ⓑ 3

Ⓒ 1

Ⓓ 2

Ⓔ -2

3. *[3 marks]*

If $f(x) = (\sqrt{x}+1)^{\frac{1}{x}+\ln x}$, $\qquad f'(x) =$

Ⓐ $\left(\sqrt{x}+1\right)^{\frac{1}{x}+\ln x}\left[\left(-\dfrac{1}{x^2}+\dfrac{1}{x}\right)\ln\left(\sqrt{x}+1\right)+\left(\dfrac{1}{x}+\ln x\right)\cdot\dfrac{1}{2(\sqrt{x}+1)}\right]$

Ⓑ $e^{\left(\frac{1}{x}+\ln x\right)\ln\left(\sqrt{x}+1\right)}\left[-\left(\dfrac{1}{x^2}+\dfrac{1}{x}\right)\ln\left(\sqrt{x}+1\right)+\left(\dfrac{1}{x}+\ln x\right)\cdot\dfrac{1}{2\sqrt{x}(\sqrt{x}+1)}\right]$

Ⓒ $\left(\sqrt{x}+1\right)^{\frac{1}{x}+\ln x}\left[\left(-\dfrac{1}{x^2}+\dfrac{1}{x}\right)\ln\left(\sqrt{x}+1\right)+\left(\dfrac{1}{x}+\ln x\right)\cdot\dfrac{1}{(\sqrt{x}+1)}\right]$

Ⓓ $e^{\left(\frac{1}{x}+\ln x\right)\ln\left(\sqrt{x}+1\right)}\left[\left(-\dfrac{1}{x^2}+\dfrac{1}{x}\right)\dfrac{\ln\left(\sqrt{x}+1\right)}{2\sqrt{x}}+\left(\dfrac{1}{x}+\ln x\right)\cdot\dfrac{1}{2x(\sqrt{x}+1)}\right]$

Ⓔ $\left(\sqrt{x}+1\right)^{\frac{1}{x}+\ln x}\left[\left(-\dfrac{1}{x^2}+\dfrac{1}{x}\right)\ln\left(\sqrt{x}+1\right)+\left(\dfrac{1}{x}+\ln x\right)\cdot\dfrac{1}{2\sqrt{x}(\sqrt{x}+1)}\right]$

4. *[3 marks]*

The slope of the tangent line to the curve $x^3 + 3y^2 = 4$ at $(1, 1)$ is

Ⓐ undefined

Ⓑ $-\dfrac{1}{3}$

Ⓒ $\dfrac{4}{3}$

Ⓓ $\dfrac{3}{4}$

Ⓔ $-\dfrac{1}{2}$

5. *[3 marks]*

On the interval $\left(\frac{1}{2}, 2\right)$, the function $f(x) = e^{-(x^2 + x^{-2})}$ has

Ⓐ a local minimum but no local maximum

Ⓑ a local maximum but no local minimum

Ⓒ a local maximum and a local minimum

Ⓓ neither a local maximum nor a local minimum

Ⓔ two local minima and no local maximum

6. *[3 marks]*

$\lim_{x \to \infty} (x^2 + 1)^{\frac{1}{x^2+2}}$

Ⓐ does not exist

Ⓑ $= e^{\frac{1}{2}}$

Ⓒ $= e$

Ⓓ $= 0$

Ⓔ $= 1$

7. *[3 marks]*

When using the Trapezoidal Rule with the interval $[-2, 6]$ divided into $n = 4$ subintervals the approximate value of

$$\int_{-2}^{6} \sqrt{1 + x^2}\, dx$$

is closest to

Ⓐ 23.04

Ⓑ 21.55

Ⓒ 22.19

Ⓓ 15.36

Ⓔ 14.37

8. *[3 marks]*

$$\int_{-1}^{1} \frac{x^{\frac{2}{3}}\ dx}{(2+x^{\frac{5}{3}})^3} =$$

Ⓐ $-\frac{1}{3}$

Ⓑ $\frac{3}{5}\ \ln 27$

Ⓒ $-\frac{64}{27}$

Ⓓ $\frac{1}{5}$

Ⓔ $\frac{4}{15}$

9. *[3 marks]*

$$\int_{1}^{e} x \ln x \, dx =$$

Ⓐ $\frac{e^2 - e + 1}{2}$

Ⓑ $\frac{1}{2}$

Ⓒ $\frac{e^2}{2}$

Ⓓ $\frac{e^2 + 1}{4}$

Ⓔ 0

10. *[3 marks]*

$$\int \frac{3x-4}{(x-1)(x-2)}\,dx =$$

Ⓐ $3\ln|(x-1)(x-2)| + C$

Ⓑ $3\ln\left|\frac{x-2}{x-1}\right| + C$

Ⓒ $3\ln\left|\frac{x-1}{x-2}\right| + C$

Ⓓ $\ln\left|(x-1)(x-2)^2\right| + C$

Ⓔ $\ln\left|\frac{x-1}{(x-2)^2}\right| + C$

11. *[3 marks]*

$$\int_8^\infty \frac{1}{\sqrt[3]{x}}\,dx$$

Ⓐ diverges

Ⓑ $= 6$

Ⓒ $= -6$

Ⓓ $= 4$

Ⓔ $= -4$

12. *[3 marks]*

If $f(x, y, z) = \dfrac{x^2 y^3}{z^4}$, $\qquad f_{xyz}(2, -1, -2) =$

Ⓐ $-\dfrac{3}{8}$

Ⓑ $-\dfrac{3}{2}$

Ⓒ $-\dfrac{1}{4}$

Ⓓ $\dfrac{3}{2}$

Ⓔ $\dfrac{3}{8}$

13. *[3 marks]*

If two products called A and B have the joint demand functions

$$q_A(p_A, p_B) = 1000 - 24p_A + 2p_A^2 - 40p_B + p_B^2$$

and

$$q_B(p_A, p_B) = 2000 + 60p_A - 5p_A^2 - 36p_B - 3p_B^2$$

then the products are competitive provided

Ⓐ $p_A < 6$ and $p_B > 20$

Ⓑ $p_A > 6$ and $p_B > 20$

Ⓒ $p_A < 6$ and $p_B < 20$

Ⓓ ($p_A < 6$ and $p_B < 20$) or ($p_A > 6$ and $p_B > 20$)

Ⓔ $p_A > 6$ and $p_B < 20$

14. *[3 marks]*

If $x(x+y+z) = yz$ then when $(x,y,z) = (1,2,3)$, $\dfrac{\partial x}{\partial z} =$

Ⓐ $\dfrac{1}{6}$

Ⓑ $\dfrac{1}{5}$

Ⓒ $\dfrac{1}{7}$

Ⓓ $\dfrac{1}{3}$

Ⓔ $\dfrac{1}{8}$

15. *[3 marks]*

If the joint demand functions for the products A and B are given by

$$q_A = \frac{4}{p_A \sqrt[3]{p_B}} \quad q_B = \frac{6}{p_B \sqrt{p_A}}$$

and the joint cost function is given by

$$C = {q_A}^2 + 2q_B$$

then when $p_A = 4$ and $p_B = 1$, $\dfrac{\partial C}{\partial p_A} =$

Ⓐ $-\frac{11}{4}$

Ⓑ $-\frac{5}{4}$

Ⓒ 2

Ⓓ -4

Ⓔ $\frac{1}{4}$

PART B. WRITTEN-ANSWER QUESTIONS

B1. *[9 marks]*

Tom owes Jerry two debts:
— \$1000 due now; and
— \$3000 plus interest at 5% compounded quarterly, due in 3 years.

They have agreed that the debts will be repaid in two payments:
— the first payment to be made in 4 years;
— the second payment to be one-third the amount of the first, to be made in 5 years.

What should the amounts of the first and second payments be, if money is worth 6% compounded semi-annually?

B2. *[11 marks]*

Find the values of x and y which minimize the function $f(x,y) = xy$ subject to the constraint $\frac{1}{x} + \frac{1}{y} = 2$ by **using Lagrange Multipliers** (no need to verify your answer is a minimum and no marks will be given for any other method).

B3. *[11 marks]*

A bakery produces oatmeal and chocolate chip cookies. It costs \$1/kg to make oatmeal cookies and \$2/kg to make chocolate chip cookies. If the bakery sells q_o kg of oatmeal cookies for \$ p_o /kg and q_c kg of chocolate chip cookies for \$ p_c /kg, then the joint demand functions are given by:

$$q_o = 100(p_c - p_o) \qquad q_c = 500 + 100(p_o - 2p_c)$$

Find the prices p_o and p_c that result in maximum profit for the bakery.

(Be sure to verify that you actually get at least a local maximum by using the second derivative test for functions of **two** variables.)

B4. *[12 marks]*

Solve the following problems showing all your work.

(a) *[6 marks]*
If $\dfrac{dy}{dx} = xe^y$ and $y = 0$ when $x = 0$, what is y when $x = 1$?

(b) *[6 marks]*
If $\dfrac{dN}{dt} = N(1 - N)$ and $N = \dfrac{1}{2}$ when $t = 1$, what is N when $t = 2$?
[You may assume that $N(t)$ is always between 0 and 1.]

B5. *[12 marks]*

In both (a) and (b), your final answer should be a number rounded to two decimal places.

(a) *[6 marks]*
Find the present value of a 2 year continuous annuity at an annual rate of 6% compounded continuously if the rate of payment at time t is $(5 - t)$ million dollars per year.

(b) *[6 marks]*
Find $\displaystyle\int_0^1 \int_1^2 (2ye^x - 5xe^y)\, dx\, dy$.

Solutions to April 2007 Exam, MAT133Y

PART A

1. ANSWER: Ⓓ

$$\left(\begin{array}{ccccc|c} 1 & 1 & 1 & 1 & 1 & 2 \\ 0 & 1 & 1 & 2 & 0 & 2 \\ 0 & 0 & 1 & 1 & 2 & 3 \end{array}\right)$$

is already in row echelon form.

$$\begin{array}{ccccc} \#\text{var} & - & \#\text{non}-\text{zero rows} & = & \#\text{parameters} \\ 5 & - & 3 & = & 2 \end{array}$$

2. ANSWER: Ⓐ

To be cont.,

$$\lim_{x\to 1} h(x) = h(1) = a - 2$$

$$\lim_{x\to 1} \frac{x^2-1}{x-1} = \lim_{x\to 1}(x+1) = 2$$

$$\text{so} \quad a - 2 = 2$$

$$a = 4$$

3. ANSWER: Ⓔ

$$\ln f = (\frac{1}{x} + \ln x)\ln(\sqrt{x}+1)$$

$$\frac{1}{f}f' = \left(-\frac{1}{x^2} + \frac{1}{x}\right)\ln(\sqrt{x}+1) + \left(\frac{1}{x} + \ln x\right)\frac{1}{\sqrt{x}+1}\cdot\frac{1}{2\sqrt{x}}$$

multiplying these by $f = (\sqrt{x}+1)^{\frac{1}{x}+\ln x}$, gets Ⓔ

4. ANSWER: Ⓔ

$$3x^2 + 6yy' = 0$$

$$y' = -\frac{x^2}{2y}$$

$$= -\frac{1}{2} \quad \text{at} \quad (1,1)$$

5. ANSWER: Ⓑ

$f'(x) = -e^{-(x^2+x^{-2})}\left[2x - \frac{2}{x^3}\right] = \frac{-2e^{-(x^2+x^{-2})}}{x^3}(x^4-1)$

$f'(x) = 0$ **only** at $x = 1$,

the **only** crit pt., on $(\frac{1}{2}, 2)$.

$$\frac{1}{2} < x < 1 \Rightarrow f' > 0 \qquad \bigwedge \qquad \text{local max at } x = 1$$
$$(1,2) \Rightarrow f' < 0$$

and that's all, so Ⓑ

6. ANSWER: Ⓔ

$$y = (x^2+1)^{\frac{1}{x^2+2}}$$
$$\ln y = \frac{\ln(x^2+1)}{x^2+2} \quad \frac{\infty}{\infty} \quad \text{as} \quad x \to \infty$$
$$\lim_{x\to\infty} \ln y = \lim_{x\to\infty} \frac{\frac{2x}{x^2+1}}{2x} = \lim_{x\to\infty} \frac{1}{x^2+1}$$
$$= 0$$
$$\ln y \to 0$$
$$y = e^{\ln y} \to e^0 = 1$$

7. ANSWER: Ⓐ

$$x_0 = -2$$
$$x_1 = 0$$
$$x_2 = 2$$
$$x_3 = 4$$
$$x_4 = 6$$

$$\triangle_x = \frac{6-(-2)}{4} = 2$$
$$T_4 = \frac{\triangle_x}{2}[y_0 + 2y_1 + 2y_2 + 2y_3 + y_4]$$
$$= 1[\sqrt{1+4} + 2\sqrt{1+0} + 2\sqrt{1+4} + 2\sqrt{1+16} + \sqrt{1+36}]$$
$$\approx 23.037 \quad \text{so} \quad Ⓐ$$

8. ANSWER: Ⓔ

Let

$$u = 2 + x^{\frac{5}{3}} \qquad x = -1 \;\Rightarrow u = 1 \qquad\qquad x = 1 \;\Rightarrow u = 3$$

$$du = \frac{5}{3} x^{\frac{2}{3}}\, dx$$

$$\begin{aligned} \text{Integral} &= \frac{3}{5} \int_1^3 \frac{du}{u^3} \\ &= \frac{\frac{3}{5}}{-2} u^{-2} \Big|_1^3 \\ &= -\frac{3}{10} \left[\frac{1}{9} - 1 \right] = \frac{3}{10} \cdot \frac{8}{9} = \frac{4}{15} \end{aligned}$$

9. ANSWER: Ⓓ

$$u = \ln x \qquad dv = x\, dx$$
$$du = \tfrac{1}{x}\, dx \qquad v = \tfrac{x^2}{2}$$

$$\begin{aligned} \text{Integ.} &= \frac{x^2}{2} \ln x \Big|_1^e - \frac{1}{2} \int_1^e x\, dx \\ &= \frac{e^2}{2} - \frac{1}{2} \frac{x^2}{2} \Big|_1^e \\ &= \frac{e^2}{2} - \frac{1}{4}(e^2 - 1) \\ &= \frac{1}{4} e^2 + \frac{1}{4} \end{aligned}$$

10. ANSWER: Ⓓ

$$\frac{3x-4}{(x-1)(x-2)} = \frac{A}{x-1} + \frac{B}{x-2}$$

$$A(x-2) + B(x-1) = 3x - 4$$

$$x = 1 : -A = -1 \;\text{ so }\; A = 1$$

$$x = 2 : \quad B = 2$$

$$\begin{aligned} \text{Integral} &= \int \left[\frac{1}{x-1} + \frac{2}{x-2} \right] dx \\ &= \ln|x-1| + 2\ln|x-2| + C \\ &= \ln(|x-1||x-2|^2) + C \end{aligned}$$

11. ANSWER: Ⓐ

$$\lim_{R\to\infty}\int_8^R x^{-\frac{1}{3}}\,dx$$
$$= \lim_{R\to\infty}\frac{3}{2}x^{\frac{2}{3}}\Big|_8^R$$
$$= \lim_{R\to\infty}\frac{3}{2}[R^{\frac{2}{3}}-4]\to\infty,\quad \text{so Ⓐ}$$

12. ANSWER: Ⓓ

$$f_x = \frac{2xy^3}{z^4}$$
$$f_{xy} = \frac{6xy^2}{z^4}$$
$$f_{xyz} = -\frac{24xy^2}{z^5}$$
$$f_{xyz}(2,-1,-2) = -\frac{24\cdot 2\cdot(-1)^2}{(-2)^5}$$
$$= \frac{48}{32} = \frac{3}{2}$$

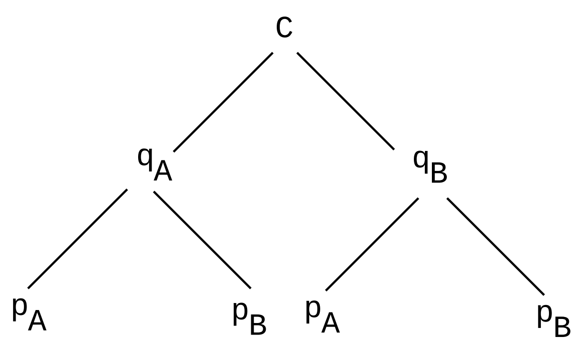
13. ANSWER: Ⓐ

$$\frac{\partial q_A}{\partial p_B} = -40+2p_B > 0 \Leftrightarrow p_B > 20$$
$$\frac{\partial q_B}{\partial p_A} = 60-10p_A > 0 \Leftrightarrow p_A < 6$$
$$\text{so}\quad p_B > 20 \quad\text{and}\quad p_A < 6$$

14. ANSWER: Ⓒ

$$\frac{\partial x}{\partial z}(x+y+z)+x(\frac{\partial x}{\partial z}+1)=y$$
$$\text{at}\ \ (1,2,3)\qquad \frac{\partial x}{\partial z}\cdot 6+\frac{\partial x}{\partial z}+1=2$$
$$7\frac{\partial x}{\partial z}=1$$
$$\frac{\partial x}{\partial z}=\frac{1}{7}$$

15. ANSWER: Ⓑ



$$\frac{\partial C}{\partial p_A} = \frac{\partial C}{\partial q_A}\frac{\partial q_A}{\partial p_A} + \frac{\partial C}{\partial q_B}\frac{\partial q_B}{\partial p_A}$$

$$\frac{\partial C}{\partial q_A} = 2q_A$$

$$= \frac{8}{p_A\sqrt[3]{p_B}}$$

$$= \frac{8}{4} = 2 \text{ at } p_A = 4,\ p_B = 1,$$

$$\frac{\partial C}{\partial q_B} = 2 \quad \text{always}$$

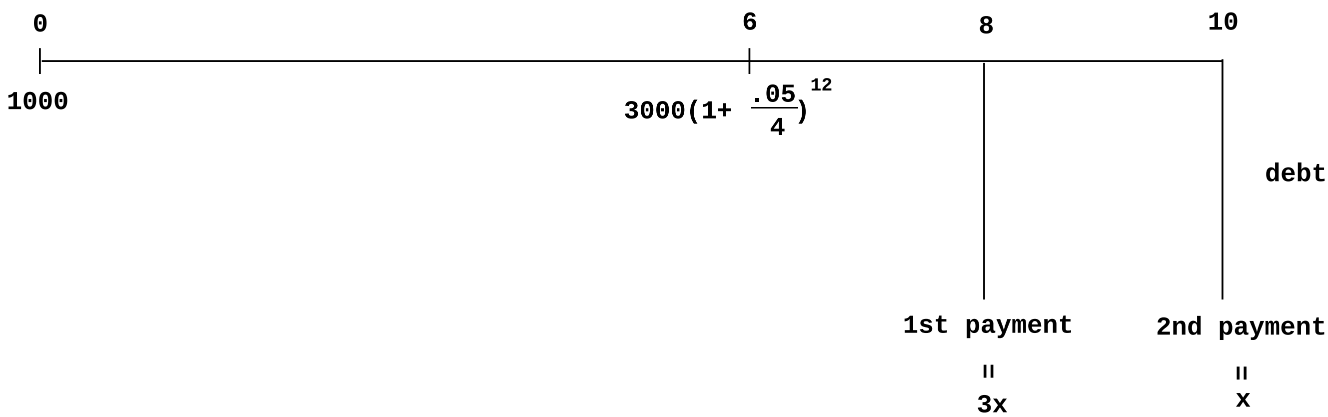
$$\frac{\partial q_A}{\partial p_A} = -\frac{4}{p_A^2\sqrt[3]{p_B}} = -\frac{4}{16} = -\frac{1}{4} \text{ at } p_A = 4, \quad p_B = 1$$

and $$\frac{\partial q_B}{\partial p_A} = -\frac{3}{p_B p_A^{3/2}} = -\frac{3}{8} \text{ at } p_A = 4, \quad p_B = 1$$

$$\frac{\partial C}{\partial p_A} = 2(-\frac{1}{4}) + 2(-\frac{3}{8}) = -\frac{5}{4}$$

PART B

B1.



We show the calculation at the end of the 5th yr (for convenience only):

$$1000(1.03)^{10} + 3000\left(1 + \frac{.05}{4}\right)^{12}(1.03)^4 = 3x(1.03)^2 + x$$

$$x = \frac{1000(1.03)^{10} + 3000\left(1 + \frac{0.5}{4}\right)^{12}(1.03)^4}{1 + 3(1.03)^2}$$

$$x = 1258.33$$

$$3x = 3775.00$$

First payment $= \$3775.00$

2nd payment $= \$1258.33$

B2.

$$\mathcal{L} = xy - \lambda(\frac{1}{x} + \frac{1}{y} - 2)$$

$$\mathcal{L}_x = y + \frac{\lambda}{x^2} = 0 \ \rightarrow x^2 y = -\lambda$$

$$\mathcal{L}_y = x + \frac{\lambda}{y^2} = 0 \ \rightarrow xy^2 = -\lambda$$

now divide the 2nd equation by the 1st :

$$\frac{x^2 y}{xy^2} = \frac{-\lambda}{-\lambda}$$

$$\frac{x}{y} = 1 \quad \text{and so} \quad x = y$$

(The division is OK because $x = 0$ and/or $y = 0$ is forbidden by the constraint $\frac{1}{x} + \frac{1}{y} = 2$, otherwise known as)

$\mathcal{L}_\lambda = 0$

But if $x = y$, $\frac{1}{x} + \frac{1}{x} = 2$ and $\boxed{x = 1, \quad \text{so} \quad y = 1}$

Note that there is a value of λ, namely $\lambda = -1$, but we don't really care about this.

B3. Let profit $= \pi$

Cost $= q_0 + 2q_c$

Revenue $= p_0 q_0 + p_c q_c$

$\pi =$ Revenue $-$ Cost

$\pi = q_0 p_0 + q_c p_c - q_0 - 2q_c = q_0(p_0 - 1) + q_c(p_c - 2)$

$\pi = 100(p_c - p_0)(p_0 - 1) + [500 + 100(p_0 - 2p_c)](p_c - 2)$

$$\frac{\partial \pi}{\partial p_c} = 100[(p_0 - 1) - 2(p_c - 2) + (5 + p_0 - 2p_c)] = 100[2p_0 - 4p_c + 8] = 0$$

$$\frac{\partial \pi}{\partial p_0} = 100[-(p_0 - 1) + (p_c - p_0) + (p_c - 2)] = 100[-2p_0 + 2p_c - 1]$$

Adding the 2 equations gives $100[-2p_c + 7] = 0$

so $p_c = 3.5$

but then using $\dfrac{\partial \pi}{\partial p_0} = 0$, $-2p_0 + 7 - 1 = 0$, so $p_0 = 3$.

The only critical point is $\boxed{p_0 = \$3.00 \quad \text{and} \quad p_c = \$3.50}$

$$\frac{\partial^2 \pi}{\partial p_c^2} = -400$$

$$\frac{\partial^2 \pi}{\partial p_0^2} = -200$$

$$\frac{\partial^2 \pi}{\partial p_0 \partial p_c} = 200 \quad \text{always}$$

$$D = \frac{\partial^2 \pi}{\partial p_c^2}\frac{\partial^2 \pi}{\partial p_c^2} - \left(\frac{\partial^2 \pi}{\partial p_0 \partial p_c}\right)^2 = (-400)(-200) - (200)^2$$

$$= 40,000 > 0$$

so our crit. pt. **is** a local extremum and $\dfrac{\partial^2 \pi}{\partial p_c^2} < 0$ so it **is** a local max.

B4.

(a)

$$\int e^{-y}\,dy = \int x\,dx$$

$$-e^{-y} = \frac{x^2}{2} + C. \quad \text{At} \quad x = 0, y = 0, \quad \text{this says}$$

$$-e^0 = 0 + C \quad \text{so} \quad C = -1$$

$$-e^{-y} = \frac{x^2}{2} - 1$$

$$\text{When} \quad x = 1, -e^{-y} = \frac{1}{2} - 1 = -\frac{1}{2}$$

$$e^{-y} = \frac{1}{2}$$

$$e^y = 2$$

$$\boxed{y = \ln 2} \approx .693$$

(b)

$$\int \frac{dN}{N(1-N)} = \int dt$$

$$\int \Big(\frac{1}{N} + \frac{1}{1-N}\Big)dN = t + C$$

$$\ln N - \ln(1-N) = t + C. \quad \text{At} \quad t = 1, N = \frac{1}{2}, \quad \text{this says}$$

$$\ln\frac{1}{2} - \ln(\frac{1}{2}) = 1 + C$$

$$C + 1 = 0 \quad \text{and} \quad C = -1$$

$$\text{Hence,} \quad \ln\frac{N}{1-N} = t - 1$$

$$\text{When} \quad t = 2, \ \ln\frac{N}{1-N} = 1$$

$$\frac{N}{1-N} = e$$

$$N = e - eN$$

$$N(1+e) = e$$

$$\boxed{N = \frac{e}{1+e}} \approx .731$$

B5.

(a)

$$\text{P.V.} = \int_0^2 (5-t)e^{-.06t}dt$$

$$u = 5-t \qquad dv = e^{-.06t}dt$$
$$du = -dt \qquad v = -\frac{e^{-.06t}}{.06}$$

$$= -\frac{(5-t)}{.06}e^{-.06t}\Big|_0^2 - \frac{1}{.06}\int_0^2 e^{-.06t}dt$$

$$= \frac{-3e^{-.12}+5}{.06} + \frac{1}{(.06)^2}e^{-.06t}\ \Big|_0^2$$

$$= \frac{5-3e^{-.12}}{.06} + \frac{e^{-.12}-1}{(.06)^2} \approx 7.576\cdots$$

so 7.58 million dollars.

(b)

$$= \int_0^1 \left[2ye^x - \frac{5x^2}{2}e^y\right]_{x=1}^{x=2} dy$$

$$= \int_0^1 \left[(2e^2y - 10e^y) - (2ey - \frac{5}{2}e^y)\right] dy$$

$$= \int_0^1 \left[(2e^2-2e)y - \frac{15}{2}e^y\right] dy$$

$$= \left[(2e^2-2e)\frac{y^2}{2} - \frac{15}{2}e^y\right]_0^1$$

$$= (e^2 - e - \frac{15}{2}e) - (-\frac{15}{2})$$

$$= \frac{15}{2} + e^2 - \frac{17e}{2}$$

$$= -8.216$$

so $\boxed{-8.22}$