University of Toronto
Department of Mathematics
Topology Examination
Friday, September 5, 2008, 1-4 p.m.
Duration: 3 hours

1. Does the fundamental group of a connected topological space X coincide with the set of homotopy classes of (unbased) maps from S^1 to X? Explain.

2. Find the De Rham cohomology of a torus $S^1 \times \cdots \times S^1$ not using Kunneth theorem.

3. Every non-orientable manifold has a 2-fold orientable cover. (You do not need to prove this).

   What is the 2-fold orientable cover of $X = RP^2$ and what is the 2-fold orientable cover of Y=Klein bottle? What are their fundamental groups? What are their universal covers?

4. (a) State the Mayer-Vietoris theorem

   (b) An open cover $\mathcal{U} = \{U_a\}$ of a topological space X is called good if each finite non-empty intersection $U_{a_1} \cap \cdots \cap U_{a_k}$ of elements of this cover is contractible. Use induction on n to show that if X admits a good open cover by n open sets, then $H_i(X) = 0$ for $i > n-1$.

5. If $f : S^n \to S^n$ has $|deg(f)| \neq 1$, prove that f has a fixed point *and* that there is a point that f carries to its antipode.