1. Does the fundamental group of a connected topological space $X$ coincide with the set of homotopy classes of (unbased) maps from $S^1$ to $X$? Explain.

2. Find the De Rham cohomology of a torus $S^1 \times \cdots \times S^1$ not using Kunneth theorem.

3. Every non-orientable manifold has a 2-fold orientable cover. (You do not need to prove this).

   What is the 2-fold orientable cover of $X = \mathbb{R}P^2$ and what is the 2-fold orientable cover of $Y = \text{Klein bottle}$? What are their fundamental groups? What are their universal covers?

4. (a) State the Mayer-Vietoris theorem

   (b) An open cover $\mathcal{U} = \{U_a\}$ of a topological space $X$ is called good if each finite non-empty intersection $U_{a_1} \cap \cdots \cap U_{a_k}$ of elements of this cover is contractible. Use induction on $n$ to show that if $X$ admits a good open cover by $n$ open sets, then $H_i(X) = 0$ for $i > n - 1$.

5. If $f : S^n \to S^n$ has $|\text{deg}(f)| \neq 1$, prove that $f$ has a fixed point and that there is a point that $f$ carries to its antipode.