1. Let $X$ and $Y$ be topological spaces. Let $A \subset X$ be closed and let $f : A \to Y$ be continuous. Define

$$Z_f := (X \amalg Y) / \sim$$

where $a \sim f(a)$ for all $a \in A$.

a) Define what it means for a topological space to be normal.

b) State Urysohn’s lemma.

c) Show that if $X$ and $Y$ are normal then $Z_f$ is normal also.

2. Let $G$ be a path-connected topological group with identity element $e$. Prove that $\pi_1(G, e)$ is Abelian.

3. Let $X$ be the outline of the tetrahedron; that is, $X = \left( \bigcup_{i=1}^{6} L_i \right) \cup \left( \bigcup_{i=1}^{4} P_i \right)$

where $L_i$ are the edges and $P_i$ are the vertices. Calculate $H_*(X; \mathbb{Z})$.

4. a) What does it mean to say that a manifold is orientable?

   Note: There is more than one possible answer to this question. Some formulations require the existence of a smooth structure on the manifold; you may assume it is a differentiable manifold if you wish.

   b) Show that real projective space $\mathbb{R}P^n$ is orientable if and only if $n$ is odd.
5. Let $T$ denote the (standard) 2-dimensional torus.
   
a) State the homology and cohomology of $T$ including the ring structure. (Just state the results; no justification is required.)
   
b) State the fundamental group of $T$ with a brief explanation of how you arrived at this answer. (Detailed proof is not required.)
   
c) Show that every map from the sphere $S^2$ to $T$ induces the zero map on cohomology.

6. a) Let $f, g : S^n \to S^n$ be continuous maps such that $f(x) \neq g(x)$ for all $x \in S^n$. Show that $f \simeq a \circ g$, where $a$ is the antipodal map.
   
b) Prove that any continuous map $f : S^{2n} \to S^{2n}$ either has a fixed point or there is a point $x$ with $f(x) = -x$.
   
c) Prove that any continuous map $f : \mathbb{R}P^{2n} \to \mathbb{R}P^{2n}$ has a fixed point.