1. Let \( f \) be an \( L^p \) function on \( \mathbb{R} \). If \( p > 4/3 \), prove that
\[
\lim_{t \to 0^+} \int_0^t x^{-1/4} f(x) \, dx = 0.
\]

2. Let \( A, B \) be measurable subsets of \([0, 1]\) with \( m(A) = m(B) = 1/4 \). For any real number \( t \), let \( B_t \) denote the translation of \( B \) by \( t \). In other words, \( B_t = \{ b + t \} \_{b \in B} \).

Prove that there exists \( t \in \mathbb{R} \) so that \( m(A \cap B_t) > \frac{1}{1000} \).

3. Give an example of a sequence of functions \( f_i \in L^2(\mathbb{R}) \) with \( \|f_i\|_{L^2} = 1 \), \( \text{supp}(f_i) \subset [0, 1] \), and with \( f_i \to 0 \) weakly in \( L^2 \). Prove that your example has all the desired properties.

4. Suppose that \( f : \mathbb{R} \to \mathbb{R} \) is a Schwartz function, and that \( |\hat{f}(\omega)| \leq 1 \) and \( |\hat{f}(\omega)| \leq |\omega|^{-4} \). Prove that \( |f(3) - f(1)| < 1000 \).