Mathematics Department
University of Toronto
Real Analysis Examination
September 2009
Duration 2 hours
No aids allowed

1. Let f be an L^p function on $\mathbb{R}$. If $p > 4/3$, prove that

$$\lim_{t \to 0^+} \int_0^t x^{-1/4} f(x) dx = 0.$$

2. Let A, B be measurable subsets of $[0,1]$ with $m(A) = m(B) = 1/4$. For any real number t, let B_t denote the translation of B by t. In other words, $B_t = \{b + t\}_{b \in B}$.

Prove that there exists $t \in \mathbb{R}$ so that $m(A \cap B_t) > \frac{1}{1000}$.

3. Give an example of a sequence of functions $f_i \in L^2(\mathbb{R})$ with $\|f_i\|_{L^2} = 1$, $supp(f_i) \subset [0,1]$, and with $f_i \to 0$ weakly in L^2. Prove that your example has all the desired properties.

4. Suppose that $f : \mathbb{R} \to \mathbb{R}$ is a Schwartz function, and that $|\hat{f}(\omega)| \leq 1$ and $|\hat{f}(\omega)| \leq |\omega|^{-4}$. Prove that $|f(3) - f(1)| < 1000$.