No Aids
Questions will be weighted equally.

1. Construct a conformal mapping (as a composition of simpler maps) from the infinite strip $|\Im z| < \frac{\pi}{4}$ onto the unit disc which takes $0$ to $0$. What is the most general such mapping?

2. Suppose that $\Omega$ is a domain in $\mathbb{C}$, $\{f_k\}$ is a sequence of analytic functions on $\Omega$, $f_k \to f$ uniformly on compact subsets of $\Omega$, and $f$ has a zero of order $N$ at $z_0 \in \Omega$. Show that there exists $\rho > 0$ such that for $k$ sufficiently large, $f_k(z)$ has exactly $N$ zeros counting multiplicities on $|z - z_0| < \rho$.

3. Evaluate
\[
\int_0^\infty \frac{\sin x}{x} dx
\]
via residues. Justify your steps. Hint: Introduce $e^{iz}$ and a suitable indented contour.