University of Toronto

Department of Mathematics

Complex Analysis Examination

Tuesday, September 2, 2008, 1–2:30 p.m.

Duration $1\frac{1}{2}$ hours

(1) Let f be an entire function such that $\mathrm{Re}(f(z)) \geq -2$ for all $z \in \mathbb{C}$. Show that f is a constant.

(2) Evaluate $\int_0^\infty e^{-x^2} \cos x^2 \, dx$ by the theory of residues.

(3) Let f be an analytic function in the unit disc $D = \{|z| < 1\}$. Suppose that $|f(z)| \leq 1$ in D. Prove that if f has at least two fixed points z_1, z_2 (that is, $f(z_i) = z_i$ for $i = 1, 2$, $z_1 \neq z_2$), then $f(z) = z$ for all $z \in D$.