University of Toronto
Department of Mathematics
Complex Analysis Examination
Tuesday, September 6, 2005, 1-2:30 p.m.
Duration 1 hour, 30 minutes

No aids allowed.
All questions are equal in value.

1. (a) Let $U, V$ denote domains in $\mathbb{C}$, and let $f : U \to V$ be a holomorphic mapping. Suppose that $f$ is proper (i.e., $f^{-1}(K)$ is compact, for every compact subset $K$ of $V$). Prove that $f(U) = V$.

(b) Is the assertion in (a) true if “holomorphic” is replaced by “continuous”? Explain.

2. Give an explicit description of the group of automorphisms of $\mathbb{C} \setminus \{0\}$ (i.e., the group of invertible holomorphic mappings from $\mathbb{C} \setminus \{0\}$ to itself).

3. Let $\{f_n\}$ be a uniformly bounded sequence of holomorphic functions on a domain $\Omega \subset \mathbb{C}$. Let $\{z_k\}$ be a sequence of distinct points in $\Omega$ with $\lim_{k \to \infty} z_k = z_0 \in \Omega$. Assume that $\lim_{n \to \infty} f_n(z_k)$ exists, for all $k$. Prove that $\{f_n\}$ converges uniformly on compact subsets of $\Omega$. 