

## Suggested prerequisites:

We recognize that our students come from many different places and with a significant range of differing backgrounds. Hence there is no fixed and rigid list of prerequisites, and applicants are considered and often admitted even if their formal previous mathematical education is very different from the informal list of prerequisites below. **In general, we'd like to see some sort of overall mathematical maturity and experience, and we appreciate (though we do not require) evidence of in-depth concentration in one mathematical discipline or another.**

Yet here's a non-binding list of courses that are recommended to applicants from within the University of Toronto in order to be seriously considered for the **doctoral stream master's program**. Students coming from other institutions will have to make the appropriate substitutions:

### **Doctoral-stream master's program:**

#### 2<sup>nd</sup> year Advanced ODE's, e.g. MAT 267H

Approximate syllabus: First-order equations. Linear equations and first-order systems. Non-linear first-order systems. Existence and uniqueness theorems for the Cauchy problem. Method of power series. Elementary qualitative theory; stability, phase plane, stationary points. Examples of applications in mechanics, physics, chemistry, biology and economics.

#### 3<sup>rd</sup> year Real Analysis, e.g. MAT 357H

Approximate syllabus: Function spaces; Arzelà-Ascoli theorem, Weierstrass approximation theorem, Fourier series. Introduction to Banach and Hilbert spaces; contraction mapping principle, fundamental existence and uniqueness theorem for ordinary differential equations. Lebesgue integral; convergence theorems, comparison with Riemann integral,  $L^p$  spaces. Applications to probability.

#### 3<sup>rd</sup> year Complex Analysis, e.g. MAT 354H

Approximate syllabus: Complex numbers, the complex plane and Riemann sphere, Möbius transformations, elementary functions and their mapping properties, conformal mapping, holomorphic functions, Cauchy's theorem and integral formula. Taylor and Laurent series, maximum modulus principle, Schwarz's lemma, residue theorem and residue calculus.

#### 3<sup>rd</sup> year Algebra, e.g. MAT 347Y

Approximate syllabus: Groups, subgroups, quotient groups, Sylow theorems, Jordan-Hölder theorem, finitely generated abelian groups, solvable groups. Rings, ideals, Chinese remainder theorem; Euclidean domains and principal ideal domains: unique factorization. Noetherian rings, Hilbert basis theorem. Finitely generated modules. Field extensions, algebraic closure, straight-edge and compass constructions. Galois theory, including insolubility of the quintic.

#### 3<sup>rd</sup> year Topology, e.g. MAT 327H

Approximate syllabus: Metric spaces, topological spaces and continuous mappings; separation, compactness, connectedness. Topology of function spaces. Fundamental group and covering spaces. Cell complexes, topological and smooth manifolds, Brouwer fixed-point theorem.

In addition to that we also value some ability in computer programming and some background in physics (though neither is required).

Likewise here's a non-binding list of courses that are recommended to applicants from within the University of Toronto in order to be seriously considered for the **terminal master's program**. Students coming from other institutions will have to make the appropriate substitutions:

**Terminal master's program:**

*Linear Algebra, e.g. MAT 224*

Approximate syllabus: Abstract vector spaces: subspaces, dimension theory. Linear mappings: kernel, image, dimension theorem, isomorphisms, matrix of linear transformation. Changes of basis, invariant spaces, direct sums, cyclic subspaces, Cayley-Hamilton theorem. Inner product spaces, orthogonal transformations, orthogonal diagonalization, quadratic forms, positive definite matrices. Complex operators: Hermitian, unitary and normal. Spectral theorem. Isometries of  $\mathbb{R}^2$  and  $\mathbb{R}^3$ .

*Groups and Symmetries, e.g. MAT 301*

Approximate syllabus: Congruences and fields. Permutations and permutation groups. Linear groups. Abstract groups, homomorphisms, subgroups. Symmetry groups of regular polygons and Platonic solids, wallpaper groups. Group actions, class formula. Cosets, Lagrange's theorem. Normal subgroups, quotient groups. Classification of finitely generated abelian groups. Emphasis on examples and calculations.

*Complex Variables, e.g. MAT 334*

Approximate syllabus: Theory of functions of one complex variable, analytic and meromorphic functions. Cauchy's theorem, residue calculus, conformal mappings, introduction to analytic continuation and harmonic functions.

*Real Analysis, e.g. MAT 337*

Approximate syllabus: Metric spaces; compactness and connectedness. Sequences and series of functions, power series; modes of convergence. Interchange of limiting processes; differentiation of integrals. Function spaces; Weierstrass approximation; Fourier series. Contraction mappings; existence and uniqueness of solutions of ordinary differential equations. Countability; Cantor set; Hausdorff dimension.