

**Quiz Four****No notes. No calculators.**

Write clearly and explain your reasoning.

Student Number:

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- 1 (10 points) Let  $\overline{G}$  denote the complement of  $G$ , and let  $n$  be the number of vertices in  $G$ . You may assume the following facts:

- (A)  $\chi(G) + \chi(\overline{G}) \leq n + 1$
- (B)  $\chi(G)q \geq n$ , where  $q$  is the size of the largest independent set in  $G$
- (C)  $\chi(G) \geq n/(n - d)$ , where  $d$  is the smallest degree of a vertex in  $G$

(a) (6 points) Show that  $\chi(G)\chi(\overline{G}) \geq n$ .

(b) (4 points) Using part (a), show that  $\chi(G) + \chi(\overline{G}) \geq 2\sqrt{n}$ .

2 (10 points)

(a) (6 points) What is the maximum number of vertices (both internal vertices and leaves) in an  $m$ -ary tree of height  $h$ ?

(b) (4 points) What is the minimum number of vertices (both internal vertices and leaves) in an  $m$ -ary tree of height  $h$ ?