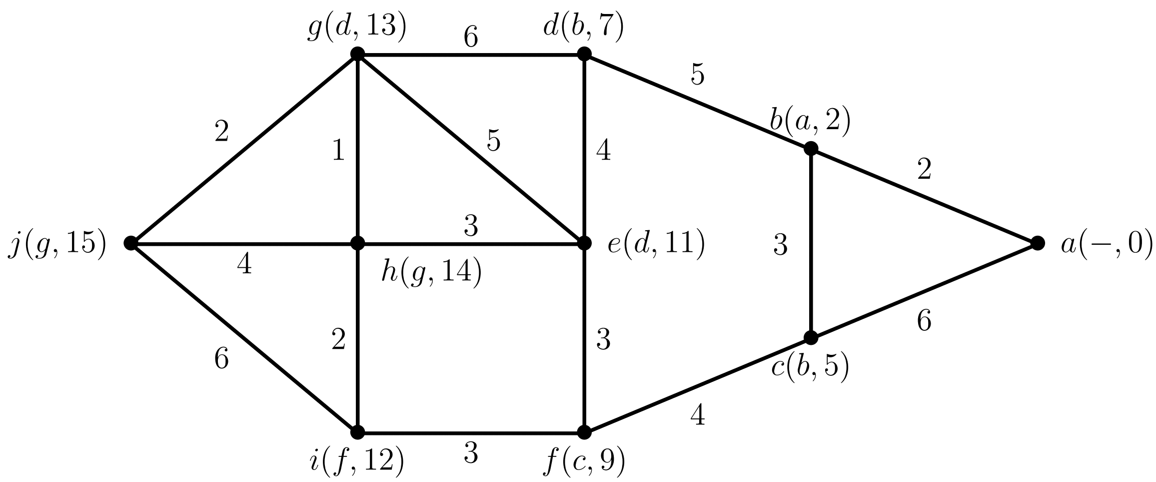
Quiz Five Solutions

1 In order to find the shortest path from vertex a to vertex j, we apply the shortest path algorithm (from Tucker, Section 4.1). That is, we label $a = (-, 0)$ and continue, at step m, to label vertices that are distance m from a. This is done in the following order:

$$\begin{aligned}
m = 2 &: \quad b = (a, 2) \\
m = 5 &: \quad c = (b, 5) \\
m = 7 &: \quad d = (b, 7) \\
m = 9 &: \quad f = (c, 9) \\
m = 11 &: \quad e = (d, 11) \\
m = 12 &: \quad i = (f, 12) \\
m = 13 &: \quad g = (d, 13) \\
m = 14 &: \quad h = (g, 14) \text{ or } h = (i, 14) \\
m = 15 &: \quad j = (g, 15)
\end{aligned}$$

We end up with the following picture:



Backtracking from j, we see that the shortest path is $a - b - d - g - j$.

2 (a) Prim's algorithm says to add the shortest edge between a vertex in T (the tree so far) and a vertex not in T. This edge is (e, f), which has a value 2 associated with it.

(b) Kruskal's algorithm says to add the shortest edge not already in T that does not form a circuit with the edges already in T. The shortest remaining edge is (j, k), which has a value 1.

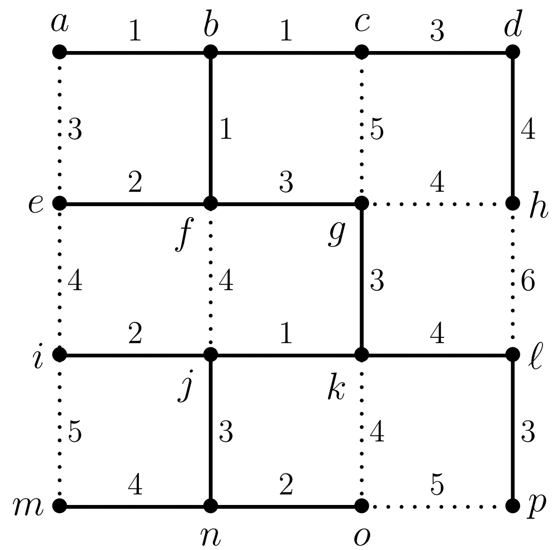
(c) (8 points) There are many correct answers to this question. We'll walk through one answer each for the two algorithms.

First, for Prim's algorithm, we add the shortest edge between a vertex in T and a vertex not in T. These are (an asterisk denotes when a choice has been made), in order:

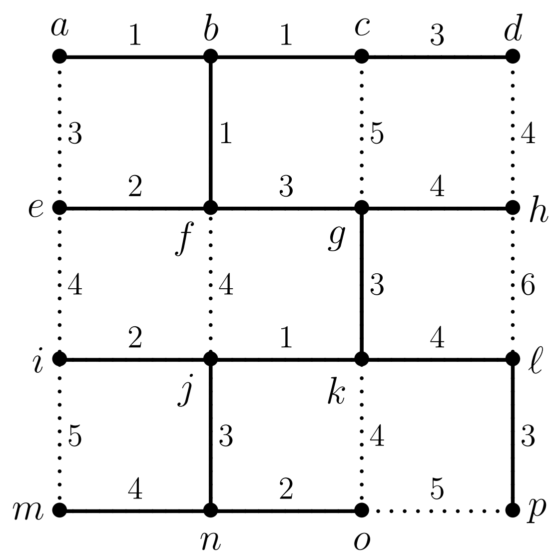
$$(e, f), \quad (f, g)^*, \quad (g, k)^*, \quad (k, j), \quad (j, i), \quad (j, n)^*,$$

(n, o), (c, d), $(d, h)^*$, $(m, n)^*$, (k, ℓ), (ℓ, p).

This produces the minimal spanning tree:



Using Kruskal's algorithm, we add edges that cost 1, then edges that cost 2, and so on, being sure to exclude edges that would create a circuit with edges already in T. Here is the resulting minimal spanning tree:



(The only choice made here is to include (g, h) rather than (d, h) when considering edges of cost 4. Thus these two are the only two minimal spanning tress.) Notice that both these trees have a "cost" of 37.