# FINAL EXAM REVIEW



**Topics:** The exam will cover all the topics, with an emphasis on the material since the midterm (which was through Section 3.3 of Tucker). The best review I can suggest is to review your notes, quizzes, and homeworks. I have included a few extra problems below, shamelessly lifted from old Math 344 classes (and other sources).
I will also have extra office hours between now and the exam; see the web page (`http://www.math.toronto.edu/garfield/344/`) for details.

**1**
(a) How many arrangements are there of the letters in the word *constitutionally*?
(b) How many arrangements are there of the letters in the word *constitutionally* so that there are no consecutive $t$'s?
(c) How many arrangements are there of the letters in the word *constitutionally* such that there are no consecutive $t$'s and the vowels occur in alphabetical order?

**2** Given any set of 100 integers show that there must be two of them such that either their sum or their difference is divisible by 197.

**3** Five men and five women each bring one book to a meeting of their bookclub. How many ways are there to distribute one book to each person so that no woman gets her own book?

**4** There are 6 people in a room some of whom are friends. Show that there must be some set of 3 of these people who are all either mutual friends or mutual strangers. Hint: suppose one of the people is Bob and divide the remaining people into those who are friends to Bob and those who are strangers to Bob.

**5** Draw two non-isomorphic graphs having 6 vertices of degrees $1, 1, 2, 2, 2, 4$ and show that any such graph is isomorphic to one of the two you have drawn.

**6**
(a) How many arrangements are there of the numbers

$$5, 5, 5, 5, 5, 4, 4, 4, 4, 3, 3, 3, 2, 2, 1$$

which have exactly 2 adjacent 5's?
(b) How many arrangements are there of the numbers

$$5, 5, 5, 5, 5, 4, 4, 4, 4, 3, 3, 3, 2, 2, 1$$

in which no 3 or 2 is adjacent to a 3 or a 2? (In case there is any confusion: this means no 3 is adjacent to either a 3 or a 2 **and** no 2 is adjacent to either a 3 or a 2.)
(c) Evaluate $\sum_{k=0}^{r}(-1)^k C(n,k)C(n,r-k)$ for any $r \leq n$ by considering the coefiicient of a certain power of $x$ in the product of two appropriate binomial expansions.

**7** A standard 6-sided cubical die with the numbers $1, 2, 3, 4, 5, 6$ on the six faces is rolled 5 times. What is the probability that the total of all the rolls is 25?

**8** Suppose $S$ is a set with 11 elements and $\mathcal{P}$ is a collection of 10 of its 4-sets. Show that there must be two sets in $\mathcal{P}$ whose intersection contains at least 2 elements. Hint: for each 2-set $E$ of $S$ let $a_E$ be the number of sets in $\mathcal{P}$ which contain $E$. What is the sum of all the $a_E$ as $E$ ranges over all the 2-sets of $S$?

**9** Draw all the non-isomorphic connected graphs with 4 or fewer vertices. No explanation is required, just a list of drawings. Your list must be such that any connected graph with 4 or fewer vertices is isomorphic to one of the graphs in your list and no two graphs in your list are isomorphic to each other.

**10** Prove that $n^5 - n$ is divisible by 10 for every positive integer $n$.

**11**
(a) How many arrangements are there of the letters *aaaaabbbbcccdde*.

(b) How many arrangements are there of the letters *aaaaabbbbcccdde* so that there are no adjacent $a$'s?

(c) How many arrangements are there of the letters *aaaaabbbbcccdde* so that no $c$ is adjacent to a $c$ or $d$ and no $d$ is adjacent to a $c$ or $d$?

**12** Given any 27 distinct numbers between 1 and 51 show that there must be 2 of these numbers which differ by 13.

**13** How many ways are there to distribute 30 pieces of candy to 4 boys and 2 girls so that no boy gets more than 5 pieces?

**14** Draw two non-isomorphic graphs having 5 vertices of degrees 1, 2, 2, 2, 3 and prove that they are non-isomorphic. Show that any such graph must be isomorphic to one of the two you have drawn.

**15** Find the number of integer solutions of the equation

$$2x + 3y + 7z = 84$$

where $x \geq 8$, $3 \leq y \leq 8$, and $0 \leq z \leq 3$.