Back to the outline of Gromov nonsqueezing.

$$\begin{tikzcd}
\text{nbhd of } B^{2n}(r) \arrow[r, hook, "\text{symp.}"'] \arrow[dr, hook] & B^2(R) \times \mathbb{R}^{2n-2} \arrow[d] \\
& (M,\omega) = \left(\mathbb{P}^1(\mathbb{C}) \times (\mathbb{R}/\lambda\mathbb{Z})^{2n-2}, R^2\omega_{FS} \oplus \omega_{std}\right)
\end{tikzcd}$$

compactify

mod out by sparse lattice

$$A = [\mathbb{P}^1(\mathbb{C}) \times \{*\}] \in H_2(M)$$

$M' = \mathbb{R}^{2n-2}/\lambda\mathbb{Z}^{2n-2}$ is symplectically aspherical, i.e. $\nexists$ a spherical homology class with symplectic area > 0. (since $\pi_2(\mathbb{R}^{2n-2}/\lambda\mathbb{Z}^{2n-2}) = 0$ ☺)

$\Longrightarrow A := [\mathbb{P}^1(\mathbb{C}) \times \{*\}] \in H_2(\mathbb{P}^1(\mathbb{C}) \times M')$ is indecomposable, i.e. $\nexists$ decomposition $A = A' + A''$ such that A', A'' are spherical with positive symplectic area.

Note: $u = (u_1, u_2) : S^2 \to M_1 \times M_2$ can be deformed such that:
- u_1 is constant on the upper hemisphere
- u_2 —— " —— lower hemisphere

Otherwise: $A' = a'[\mathbb{P}^1(\mathbb{C}) \times \{*\}] + [\{*\} \times C']$

$A'' = a''[\mathbb{P}^1(\mathbb{C}) \times \{*\}] + [\{*\} \times C'']$

$a', a'' \in \mathbb{Z}$, $a' + a'' = 1$, WLOG: $a'' \leq 0$

$$\underbrace{\text{s.area}(C'')}_{\substack{\text{cannot be pos.}\\ \text{as } M' \text{ is symp.}\\ \text{aspherical}}} = \underbrace{\text{s.area}(A'')}_{\substack{\text{cannot be}\\ \text{positive}}} \underbrace{- a''\pi R^2}_{\geq 0}$$

$\Longrightarrow$

Next, for (M,ω) compact, $A \in H_2(M)$,

$$\mathcal{M}_A = \left\{ (f, J) \;\middle|\; J \in \mathcal{J}(M,\omega),\ f : \mathbb{P}^1(\mathbb{C}) \to M \text{ } J\text{-hol.},\ [f] = A \right\}$$

Gromov compactness:

If $J_n \to J_\infty$ in $\mathcal{J}(M,\omega)$ and $(f_n, J_n) \in \mathcal{M}_A$

with $\displaystyle\int_{\mathbb{P}^1(\mathbb{C})} f_n^* \omega = \omega(A)$ all the same $\to$ bounded, then

$\exists$ subsequence that "weakly converges to a cusp curve"

$$\bigcup_j f_\infty^{(j)} : \mathbb{P}^1(\mathbb{C}) \to M \quad \xrightarrow[J_\infty\text{-hol.}]{} M$$

"bubble tree"

We obtain $A = \sum_j \underbrace{[f_\infty^{(j)}]}_{\substack{\text{spherical class} \\ \text{of area } > 0}}$

Thus if A is indecomposable, there can be only one j!

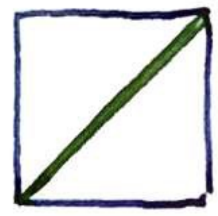
Conclusion If A is ~~indecomposable~~ indecomposable $\Rightarrow$

$\exists h_n \in PSL_2(\mathbb{C})$ such that

$$f_n \circ h_n \xrightarrow[n \to \infty]{} f_\infty : \mathbb{P}^1(\mathbb{C}) \to M$$
$$[f_\infty] = A$$

E.g. When A is decomposable, what follows is a weak convergence to a cusp curve with two j's:

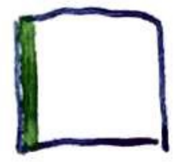
$$\mathbb{P}^1(\mathbb{C}) \xrightarrow{\text{Diag}} \mathbb{P}^1(\mathbb{C})^{\times 2}$$



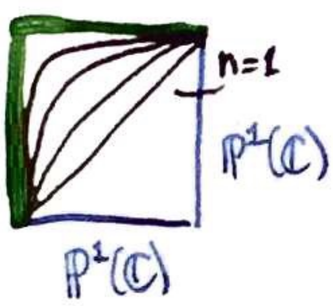
$$A = [\mathbb{P}^1(\mathbb{C}) \times \{*\}] + [\{*\} \times \mathbb{P}^1(\mathbb{C})]$$

$$\mathbb{P}^1(\mathbb{C}) = \mathbb{C} \sqcup \{\infty\}, \quad f_n(z) = \left(\tfrac{z}{n}, z\right)$$

As $n \to \infty$, $f_n(z) \to (0, z)$



$$f_n \circ h_n(z) = (z, nz), \quad f_n \circ h_n \to \{z \mapsto (z, \infty)\}$$



The weak limit:

$\{0\} \times \mathbb{P}^1(\mathbb{C})$
$\mathbb{P}^1(\mathbb{C}) \times \{\infty\}$

Thus
$$\mathcal{M}_A / PSL_2(\mathbb{C}) \xrightarrow{\pi} \mathcal{J} \quad \text{is proper}$$
$$(f, J) \longmapsto J$$
if A is indecomposable.

$$\Rightarrow \frac{\mathcal{M}_A \times \mathbb{P}^1(\mathbb{C})}{PSL_2(\mathbb{C})} \xrightarrow{\pi \times ev} \mathcal{J} \times M$$
$$((f, J), z) \longmapsto (J, f(z))$$

is proper if A is indecomposable.

BIG LEMMA $\Rightarrow$ $\pi \times ev$ is onto in the case we care about.

Choose J such that ψ is holomorphic

$$\begin{tikzcd} N \arrow[r, "\varphi"] \arrow[d, phantom, "\cap" rotate=90] & B^{2n}(r) \arrow[d, "\psi"', "\substack{\text{holomorphic} \\ \text{symp. embedding}}"] \\ \mathbb{P}^1(\mathbb{C}) \arrow[r, "f"', "\substack{J\text{-hol. thru} \\ \psi(0)\ \exists \text{ by big lem.}}"] & (M,\omega,J) \end{tikzcd}$$

φ is a proper hol. curve through ~~th~~ 0 in $B^{2n}(r)$.

$$\pi r^2 \underset{\substack{\uparrow \\ \text{Wirtinger}}}{\leq} S.\text{area}(\varphi) \leq S.\text{area}(f) = \omega(A) = \pi R^2$$

$$\frac{\mathcal{M}_A \times \mathbb{P}^1(\mathbb{C})}{PSL_2(\mathbb{C})} \xrightarrow{\pi \times ev} \mathcal{J}_\xi \times M$$

<u>Morally</u> these are manifolds.

$\mathcal{J} \times M$ is connected.

$J_0 \in \mathcal{J}$ the standard complex structure on $\mathbb{P}^1(\mathbb{C}) \times M'$

$\pi^{-1}(J_0) = \{(f, J_0)\}$. $f : \mathbb{P}^1(\mathbb{C}) \longrightarrow \mathbb{P}^1(\mathbb{C}) \times M'$
$z \longmapsto (h(z), *)$

$h \in PSL_2(\mathbb{C})$

So the preimage of $(J_0, *)$ in $\frac{\mathcal{M}_A \times \mathbb{P}^1(\mathbb{C})}{PSL_2(\mathbb{C})}$

contains only one point.

Finish by an ∞-dimensional analogue of:

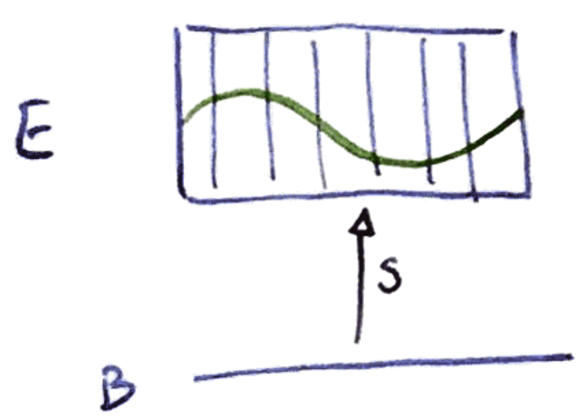
$$\boxed{\text{For } \psi : W \to N \text{ smooth, } W, N \text{ mflds, connected and } \psi \text{ proper, "degree theory" implies if } \exists \text{ pt in } N \text{ which is } \psi\text{-regular, and whose preimage is a singleton in } W \text{, then } \psi \text{ is } \underline{\text{onto}}.}$$

Office Hour

Consider a group action on a mfld $G \circlearrowright B$ which lifts to a bundle

$$
\begin{array}{ccc}
G & \circlearrowright & E \\
 & & \downarrow \\
G & \circlearrowright & B
\end{array}
$$

We use this to build an action $G \circlearrowright \Gamma(B,E)$



For $s \in \Gamma(B,E)$, $a \in G$

$$\boxed{(a.s)(b) := a(s(a^{-1}(b)))}$$

$G \circlearrowright M \Rightarrow G \circlearrowright C^{\infty}(M)$

$a \in G \qquad a.f := f \circ a^{-1}$