I have been working on a joint project with my supervisor Yael Karshon and colleague Jeffrey Carlson. Recall that a stable complex structure on a manifold M is represented by a fibrewise complex structure on the sum $TM \oplus \mathbb{R}^k$ of the tangent bundle with a trivial real vector bundle. Two such fibrewise complex structures on $TM \oplus \mathbb{R}^k$ and $TM \oplus \mathbb{R}^l$ represent the same stable complex structures on M if there is a complex linear bundle isomorphism $TM \oplus \mathbb{R}^k \oplus \mathbb{R}^a \simeq TM \oplus \mathbb{R}^l \oplus \mathbb{R}^b$ (for some $a, b \geq 0$). Let $T \simeq (S^1)^n$ act on a compact stable complex oriented manifold M with isolated fixed points. The Atiyah-Bott localisation formula says that if α is an equivariantly closed form, then $\int_M \alpha = \sum_{p \in M^T} \frac{\alpha_p}{\tilde{e}(\nu_p M)}$; in particular, integrating mixed characteristic classes of degree less than the dimension of M results in some expressions involving the weights of the torus action at the fixed points which must vanish.

In our project, we investigate to what extent the converse holds. Explicitly, let $m \in \mathbb{N}$, and suppose we are given a finite set X whose elements are pairs $p = (L_p, \epsilon_p)$ where L_p is a set of m torus weights, and $\epsilon_p \in \{\pm 1\}$. We ask whether we can always construct a compact oriented manifold M together with a torus action $T \times M \to M$ and a T-equivariant stable complex structure, such that the fixed points of the torus action are in bijection with the set S, with L_p being the set of weights of the torus action on the tangent space at the fixed point p, and such that $\epsilon_p = +1$ if and only if the orientation on T_pM induced by the stable complex structure agrees with the given orientation on M.

We prove – via explicit construction – that this is indeed possible for 4-manifolds, under the assumption that the torus action is *locally standard* (that is, the dimension of the torus is half the dimension of the manifold, and at each fixed point p the set of weights of the torus action on T_pM forms a $\mathbb{Z}$-basis for the weight lattice). We relate our work to that of Alistair Darby on the same topic.

In future work, we will consider the problem under other simplifying assumptions – that the action is a circle action; that the action is free away from the fixed points; that the stabilisers at each point are all connected – before turning our attention to the general case.