This assignment is due on Tuesday March 26th at the beginning of class. You may either handwrite this assignment or typeset it using LaTeX; either way please submit a .pdf file through UTORsubmit. Please also submit a hard copy in class.

Please read the chapter "The isoperimetric problem" from George Polya's "Mathematics and plausible reasoning", Volume I. Discuss the following problems with your friends, but write the solutions on your own.

1. (Adapted from Question 14 on page 183 of Polya's chapter.) Point out the differences, if any, between the following two questions.
   - Consider curves with a given perimeter. If C is such a curve, but not the circle, then we can construct another curve C' that encloses a greater area; such a construction is carried out in exercises in Polya's chapter. Can we conclude that the circle encloses the greatest area?
   - Consider positive integers. If n is such an integer, but not 1, then we can construct another integer n' greater than n by setting $n' = n^2$. (The condition $n > 1$ is essential; our construction $n' = n^2$ failes for $n = 1$ as $1^2 = 1$.) Can we conclude that 1 is the greatest integer?

2. (Adapted from Question 7 on page 182 of Polya's chapter.) Use the method of Polya's section 8 to sketch a proof that the following two statements are equivalent without proving either one of these statements.
   - Of all boxes with a given surface area, the cube has the maximal volume.
   - Of all boxes with a given volume, the cube has the minimal surface area.

3. Polya's Table II provides evidence that a circular membrane has a lower principal frequency than membranes of other shapes with the same area. How does this differ from the "evidence", described in section 2 of Polya's chapter, that the wood from some particular tree has the least specific weight among all existing kinds of trees?

4. (Adapted from Question 9 on page 182 of Polya's chapter.) Choose a special case of the following problem, state it, and solve it.

   > You are given a curve C, which is not a circle, that has length L and encloses area A. Construct a curve C' with the same length but which encloses an area A' that is larger than A.

5. What does roundness of soap bubbles have to do with the "isoperimetric theorem in space"?