Assignment 2

This assignment is due on Tuesday January 29th at the beginning of class. You may either handwrite this assignment or typeset it using LaTeX; either way please submit a .pdf file through UTORsubmit. Please also submit a hard copy in class.

1. Explain this joke:

   `http://xkcd.com/1153/`

   (Hints: (1) look up the "arrow paradox"; (2) place the mouse cursor on the picture.)

2. Use quantifiers ($\forall$, $\exists$), without negations, to give a formal definition of what it means for
   a) A sequence (x_n) to diverge
   b) A series $\sum_{n=0}^{\infty}$ to converge

3. Prove directly from your definition that the series $\sum_{n=0}^{\infty} 2^n$ diverges.

4. Prove that if $\sum_{n=0}^{\infty} x_n$ converges then $x_n \to 0$. Is the converse true?

5. Prove that if the series $\sum_{n=0}^{\infty} x_n$ is absolutely convergent, then it converges.

6. Let $\sim$ be a binary relation that satisfies the following two properties.

   (1) for all x, y, z, if $x \sim z$ and $y \sim z$ then $x \sim y$.
   (2) for all x, we have $x \sim x$.

   Prove that $\sim$ is an equivalence relation.