FUNDAMENTAL CONCEPTS IN DIFFERENTIAL GEOMETRY
FALL 2000
EXERCISES HANDOUT # 7

1. Exercises for the Proper Course

1. Criticize the following "counter example" of Sard's theorem. Let M^0 be the real line with the discrete topology. The canonical map $M^0 \to \mathbb{R}$ has no regular values.

2. Let $\gamma : \mathbb{R} \to \mathbb{R}^2$ be a smooth curve in the plane. Let K be the set of all $r \in \mathbb{R}$ such that the circle of radius r about the origin is tangent to γ at some point. Show that K has an empty interior in $\mathbb{R}$.

3. A "probability vector" is a vercor in $\mathbb{R}^n$ whose coordinates are all non-negative and add up to 1. A"stochastic matrix" is an $n \times n$ matrix whose columns are probability vectors. Prove that every stochastic matrix A has a fixed probability vector; i.e., a probability vector v such that $Av = v$.

4. Prove that every compact 1-manifold with boundary is a finite disjoint union of circles and closed intervals.

5. Show that every matrix with positive real entries has a positive eigenvalue.

6. Show that $\mathbb{R}^n$ is not homeomorphic to $\mathbb{R}^m$ for $m \neq n$. (Note: I really mean "not homeomorphic" and not just "not diffeomorhic").

Date: 12 Dec., 2000.