FUNDAMENTAL CONCEPTS IN DIFFERENTIAL GEOMETRY
FALL 2000
HANDOUT # 4

1. Exercises for the Proper Course

1. Let

$$O(m,n) = \{A \in M_{m+n}(\mathbb{R})\colon\, B(Ax, Ay) = B(x,y) \ \ \forall x, y \in \mathbb{R}^{m+n}\}$$

where B is the bilinear form

$$B(x,y) = \sum_{i=1}^{m} x_i y_i - \sum_{j=m+1}^{m+n} x_j y_j .$$

(a) Show that $O(m,n)$ is a group.
(b) Show that $O(m,n)$ is a smooth manifold.
(c) Explain why $O(m,n)$ is a Lie group.
(d) Describe the tangent space to $O(m,n)$ at I.

2. Consider the real valued function $f(x,y,z) = (2-(x^2+y^2)^{\frac{1}{2}})^2 + z^2$ defined on $\mathbb{R}^3 - \{(0,0,z)\}$.

(a) Show that 1 is a regular value of f. Identify the manifold $M = f^{-1}(1)$.
(b) Show that M is transverse to the manifold

$$N = \{(x,y,z) \in \mathbb{R}^3 : x^2 + y^2 = 4\}.$$

Identify the manifold $M \cap N$.

(c) Show that M is *not* transverse to the surface

$$N = \{(x,y,z) \in \mathbb{R}^3 : x^2 + y^2 = 1\}.$$

Is $M \cap N$ a manifold ?

(d) Show that M is *not* transverse to the plane

$$N = \{(x,y,z) \in \mathbb{R}^3 : x = 1\}.$$

Is $M \cap N$ a manifold ?

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