

I Understand Khovanov Homology

The Jones polynomial:

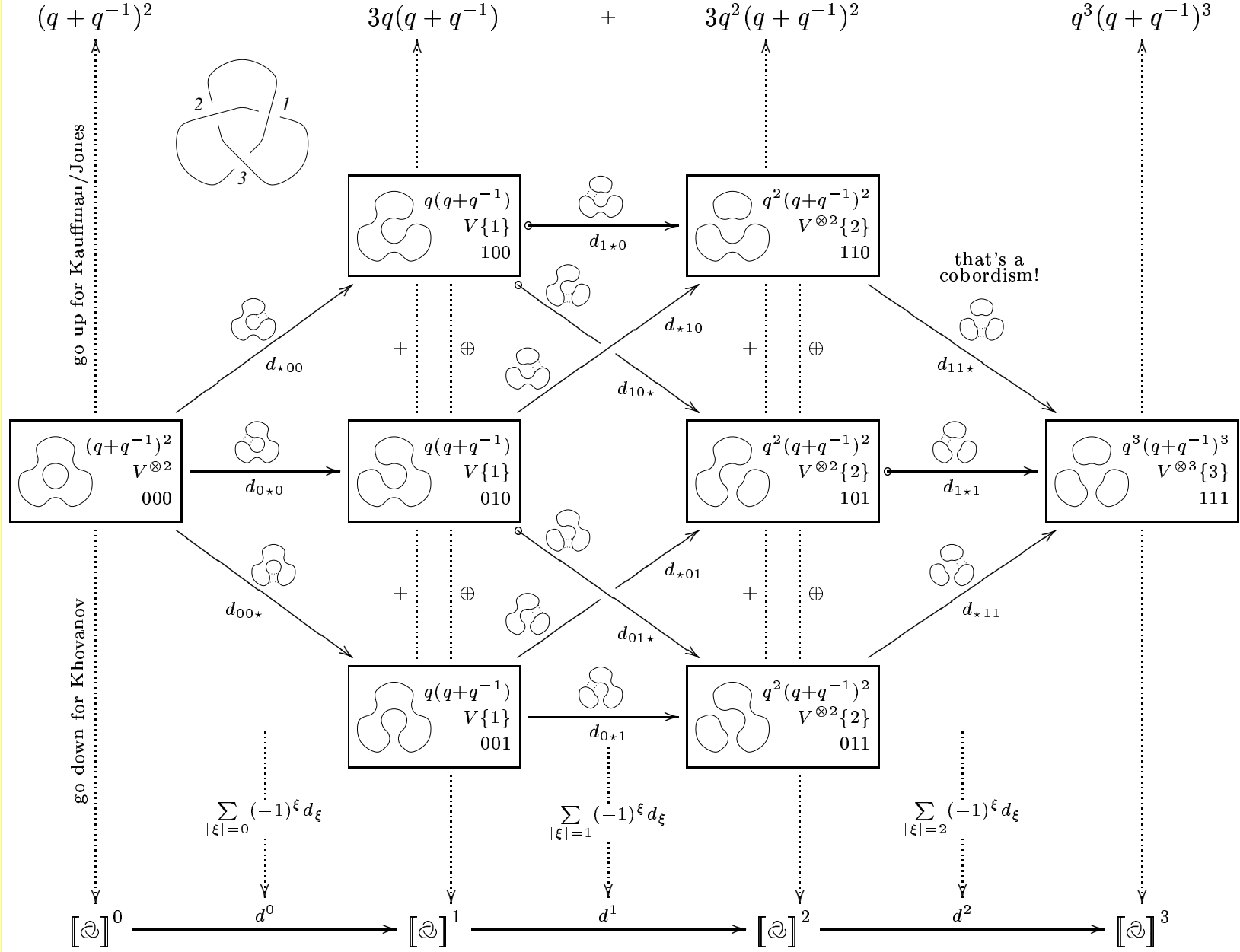
Definition. $\hat{J} : \mathcal{K} \mapsto q \langle -q^2 \smile \rangle, \quad \hat{J} : \mathcal{K} \mapsto -q^{-2} \smile + q^{-1} \langle \rangle,$

valued in $\text{king-free tangles mod } \bigcirc = q + q^{-1}$

Invariance under R2:

$$\begin{aligned} \hat{J} : \left(\begin{array}{c} \diagup \\ \diagdown \end{array} \right) &\mapsto -q^{-1} \left(\begin{array}{c} \diagup \\ \diagdown \end{array} \right) + \left(\begin{array}{c} \diagdown \\ \diagup \end{array} \right) + \left(\begin{array}{c} \diagdown \\ \diagdown \end{array} \right) - q \left(\begin{array}{c} \diagup \\ \diagup \end{array} \right) \\ &= -q^{-1} \langle \smile \rangle + \langle \smile \rangle + \langle \smile \rangle - q \langle \smile \rangle \\ &= \langle \smile \rangle \end{aligned}$$

Example / Khovanov Homology:



Who cares?



Computable!

```

In[1]:= AppendTo[$Path, "C:/drorbn/projects/KAtlas/"];
<< KnotTheory`
Loading KnotTheory` (version of September 14, 2005, 13:37:36)...

In[3]:= Kh[Knot[3, 1], Modulus -> 0][q, t]

KnotTheory::loading: Loading precomputed data in PD4Knots`.

Out[3]:= 1/q^3 + 1/q + 1/q^9 t^3 + 1/q^5 t^2
    
```

Witten: Everything in knot theory comes from physics
Drinfel'd: Everything in knot theory comes from quantum groups

HOMFLY-PT:

$$aH(\smile) - a^{-1}H(\smile) = zH(\langle \rangle); \quad H(\bigcirc) = 1$$

$$q^n P_n(\smile) - q^{-n} P_n(\smile) = (q - q^{-1}) P_n(\langle \rangle); \quad P_n(\bigcirc) = \frac{q^n - q^{-n}}{q - q^{-1}}$$

Conjecture: (I. Frenkel, though he may disown this version)

1. Every object in mathematics is the Euler characteristic of a complex.
2. Every operation in mathematics lifts to an operation between complexes.
3. Every identity in mathematics remains true up to homotopy.

I. Frenkel



"God created the knots, all else in topology is the work of mortals"

Leopold Kronecker (modified)

