Dror Bar-Natan: Talks: MSRI-0808: Projectivization, w-Knots, Kashiwara-Vergne and Alekseev-Torossian:

We Mean Business

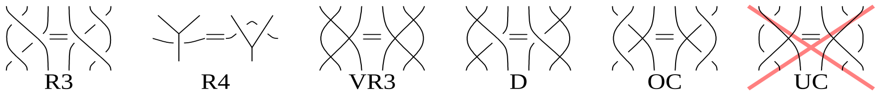
Trivalent (framed) w-tangles:

further operations: delete, unzip.

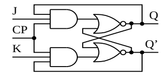
$wTT = CA\langle$  $\rangle$ / R123, R4 (for vertices), F, OC.

$= PA\langle$  $\rangle$ / R1234, F, VR1234, D, OC.

(=tangles in thick surfaces, modulo stabilization)

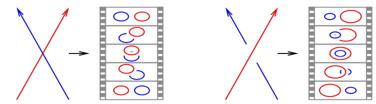


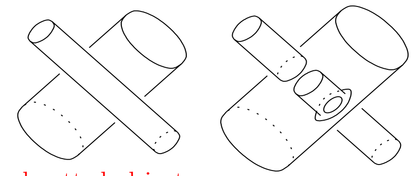
Circuit Algebras



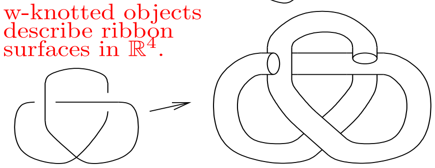
* Have "circuits" with "ends"
* Can be wired arbitrarily.
* May have "relations" - de-Morgan, etc.

w-braids describe flying rings:



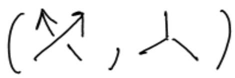
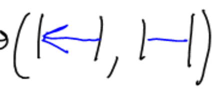


w-knotted objects describe ribbon surfaces in $\mathbb{R}^4$.

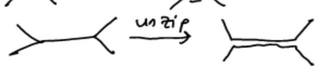


(and π_1 is preserved)

Partial Dictionary.

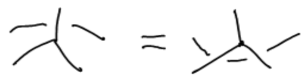
$(R, F) \longleftrightarrow ($  $)$ $(r, t) \longleftrightarrow ($  $)$

$R^{12}R^{13}R^{23} = R^{23}R^{13}R^{12} \longleftrightarrow$ 

$FF^! = I \longleftrightarrow$ 

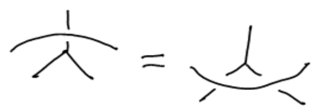
$F^{-1}e(x+y)F = e(x)e(y)$

⇕

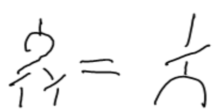
$F^{23}R^{1,23} = R^{12}R^{13}F^{23} \longleftrightarrow$ 

$R^{12,3} = R^{13}R^{23}$

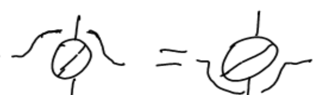
⇕

$F^{12,3}R^{12,3} = R^{13}R^{23}F^{12,3} \longleftrightarrow$ 

(unforbidding FI makes this automatic)

$RF^{21}e(-t) = F \longleftrightarrow$ 

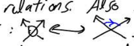
$\Phi = (F^{12,3})^{-1}(F^{1,2})^{-1}F^{2,3}F^{1,23} \longleftrightarrow$  = "The tetrahedron"

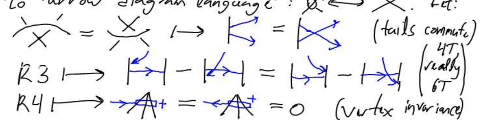
$\Phi \in$ sdr $\longleftrightarrow$ 

The pentagon and the hexagons follow, with a minor twist, from the fact that we have an unzip-behaved invariant of KTG's.

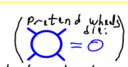
For the Experienced
(and sharp-eyed)

The "Chord Diagrams" — $\mathcal{A}_n^{wt}$. As we did for quandles, substitute

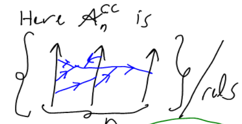
into the various moves, to get relations. Also switch to "arrow diagram language":  Get:



The "Jacobi Diagrams" — $\mathcal{A}_n^{cc}$.
Theorem. $\mathcal{A}_n^{wt}$ is $\mathcal{A}_n^{cc}$ is $\mathcal{U}(\mathrm{tder}_n)$.



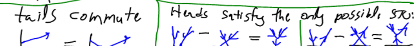
Here $\mathcal{A}_n^{cc}$ is



trivalent directed trees with only 2-in 1-out vertices

In tensorland, this is "Co-commutative Lie-bialgs"

Rels: tails commute | Heads satisfy the only possible STU:



+also IHX and vertex invariance

The Map $\alpha\colon \mathcal{A}_n^{tree} \to \mathcal{A}_n^{cc}$:

Theorem. α is an injection on $\mathcal{A}_n^{tree} \cong \mathcal{U}(\mathrm{sder}_n)$. Furthermore, there is a simple charactarization of im α, so we can tell "an arrowless element" when we see it.

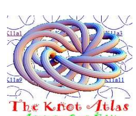
The Main Theorem. (approximate, false as stated) F's in Sol_0^τ are in a bijective correspondance with tree-level associators for ordinary paranthesized tangles (or ordinary knotted trivalent graphs) / with homomorphic expansions for trivalent w-tangles / with solutions of the Kashiwara-Vergne problem.

Extra. Restricted to knots, we get precisely the Alexander polynomial.

Disclaimer. Orientations, rotation numbers, framings, the vertical direction and the cyclic symmetry of the vertex may still make everything uglier. I hope not.

"God created the knots, all else in topology is the work of mortals"

Leopold Kronecker (paraphrased)



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