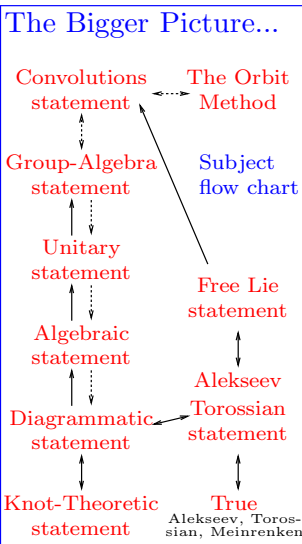




The Bigger Picture...

u-knots, v-knots, w-knots, virtual knots, links, crossings, Reidemeister moves, etc.



What are w-Trivalent Tangles?

$\{\text{knots} \& \text{links}\} = \text{PA} \langle \text{crossings} \mid R123 : \text{crossing} = \text{crossing} \rangle_{0 \text{ legs}}$

$\{\text{trivalent tangles}\} = \text{PA} \langle \text{trivalent crossings} \mid R23, R4 : \text{trivalent crossing} = \text{trivalent crossing} \rangle$

wTT = $\{\text{trivalent w-tangles}\} = \text{PA} \langle \text{w-generators} \mid \text{w-relations} \mid \text{w-unary w-operations} \rangle$

The w-generators.

www.math.toronto.edu/~drorbn/Talks/KSU-090407

Homomorphic expansions for a filtered algebraic structure \mathcal{K} :

$$\text{ops} \curvearrowright \mathcal{K} = \mathcal{K}_0 \supset \mathcal{K}_1 \supset \mathcal{K}_2 \supset \mathcal{K}_3 \supset \dots$$

$$\text{ops} \curvearrowright \text{gr } \mathcal{K} := \mathcal{K}_0/\mathcal{K}_1 \oplus \mathcal{K}_1/\mathcal{K}_2 \oplus \mathcal{K}_2/\mathcal{K}_3 \oplus \mathcal{K}_3/\mathcal{K}_4 \oplus \dots$$

An **expansion** is a filtration respecting $Z : \mathcal{K} \rightarrow \text{gr } \mathcal{K}$ that "covers" the identity on $\text{gr } \mathcal{K}$. A **homomorphic expansion** is an expansion that respects all relevant "extra" operations.

A **Ribbon 2-Knot** is a surface S embedded in \mathbb{R}^4 that bounds an immersed handlebody B , with only "ribbon singularities"; a ribbon singularity is a disk D of transverse double points, whose preimages in B are a disk D_1 in the interior of B and a disk D_2 with $D_2 \cap \partial B = \partial D_2$, modulo isotopies of S alone.

Filtered algebraic structures are cheap and plenty. In any \mathcal{K} , allow formal linear combinations, let \mathcal{K}_1 be the ideal generated by differences (the "augmentation ideal"), and let $\mathcal{K}_m := \langle (\mathcal{K}_1)^m \rangle$ (using all available "products").

The **w-relations** include R234, VR1234, M, Overcrossings Commute (OC) but not UC, $W^2 = 1$, and **funny interactions** between the wen and the cap and over- and under-crossings:

"An Algebraic Structure"

- Has kinds, objects, operations, and maybe constants.
- Perhaps subject to some axioms.
- We always allow formal linear combinations.

OC: $\text{crossing} \leftrightarrow \text{crossing}$ as $\text{ribbon crossing} \leftrightarrow \text{ribbon crossing}$ (yet not UC)

Challenge. Do the Reidemeister!

Example: Pure Braids. PB_n is generated by x_{ij} , "strand i goes around strand j once", modulo "Reidemeister moves". $A_n := \text{gr } PB_n$ is generated by $t_{ij} := x_{ij} - 1$, modulo the 4T relations $[t_{ij}, t_{ik} + t_{jk}] = 0$ (and some lesser ones too). Much happens in A_n , including the Drinfel'd theory of associators.

The unary w-operations

Our case(s).

$$\mathcal{K} \xrightarrow{Z: \text{high algebra}} \mathcal{A} := \text{gr } \mathcal{K} \xrightarrow{\text{given a "Lie" algebra } \mathfrak{g}} \mathcal{U}(\mathfrak{g})$$

solving finitely many equations in finitely many unknowns

low algebra: pictures represent formulas

\mathcal{K} is knot theory or topology; $\text{gr } \mathcal{K}$ is finite combinatorics: bounded-complexity diagrams modulo simple relations.

[1] <http://qlink.queensu.ca/~4lb11/interesting.html> 29/5/10, 8:42am

Also see <http://www.math.toronto.edu/~drorbn/papers/WKO/>

Just for fun.

$\mathcal{K} = \left\{ \text{images of people} \right\} = \left(\text{The set of all b/w 2D projections of reality} \right)$

$\mathcal{K}/\mathcal{K}_1 \leftarrow \mathcal{K}/\mathcal{K}_2 \leftarrow \mathcal{K}/\mathcal{K}_3 \leftarrow \mathcal{K}/\mathcal{K}_4 \leftarrow \dots$

crop rotate Adjoin

An expansion Z is a choice of a "progressive scan" algorithm.

crop rotate adjoin

$\mathcal{K}/\mathcal{K}_1 \oplus \mathcal{K}_1/\mathcal{K}_2 \oplus \mathcal{K}_2/\mathcal{K}_3 \oplus \mathcal{K}_3/\mathcal{K}_4 \oplus \mathcal{K}_4/\mathcal{K}_5 \oplus \mathcal{K}_5/\mathcal{K}_6 \oplus \dots$

$\mathbb{R} \quad \parallel \quad \ker(\mathcal{K}/\mathcal{K}_4 \rightarrow \mathcal{K}/\mathcal{K}_3)$