1. $\mathrm{proj}\,\mathcal{K}^w(\uparrow_n) \cong_j \mathcal{U}\left((\mathfrak{a}_n \oplus \mathfrak{tder}_n) \ltimes \mathfrak{tr}_n\right)$

— *All Signs Are Wrong!* —

I understand Drinfel'd and Alekseev−Torossian, I don't understand Etingof−Kazhdan yet, and I'm clueless about Kontsevich

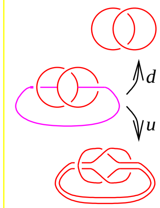
Dror Bar−Natan, Montpellier, June 2010, http://www.math.toronto.edu/~drorbn/Talks/Montpellier−1006/

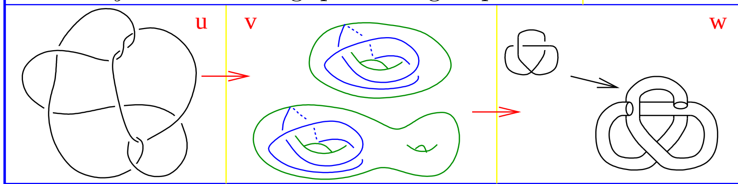
Cans and Can't Yets.

(arbitrary algebraic structure) $\xrightarrow[\text{machine}]{\text{projectivization}}$ (a problem in graded algebra)

www.katlas.org

- Feed knot-things, get Lie algebra things.
- (u-knots)→(Drinfel'd associators).
- (w-knots)→(K-V-A-E-T).
- Dream: (v-knots)→(Etingof-Kazhdan).
- Clueless: (???)→(Kontsevich)?
- Goals: add to the Knot Atlas, produce a working AKT and touch ribbon 1-knots, rip benefits from *truly* understanding quantum groups.

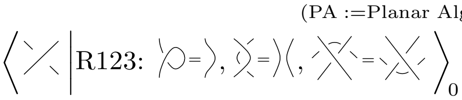




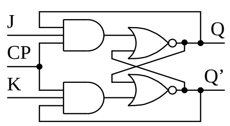
u-Knots

(PA :=Planar Algebra)

$\left\{\begin{matrix}\text{knots}\\ \&\text{links}\end{matrix}\right\} = \mathrm{PA}\left\langle \;\middle|\; \text{R123: } \dots \right\rangle_{0\text{ legs}}$



Circuit Algebras



A J-K Flip Flop

Infineon HYS64T64020HDL-3.7-A 512MB RAM

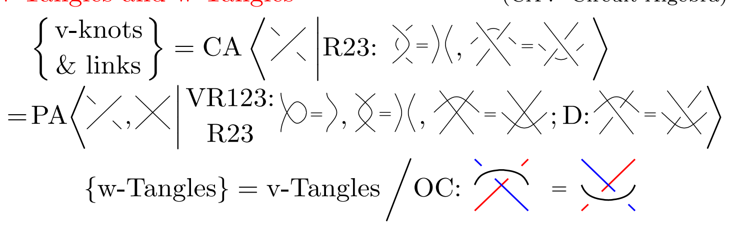
v-Tangles and w-Tangles

(CA :=Circuit Algebra)

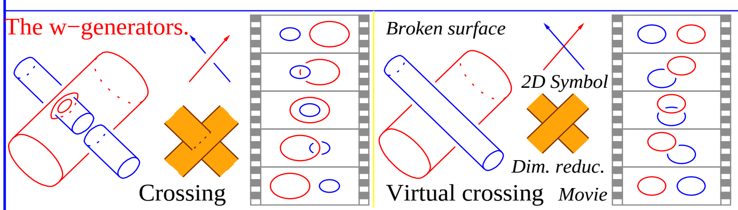
$\left\{\begin{matrix}\text{v-knots}\\ \&\text{ links}\end{matrix}\right\} = \mathrm{CA}\left\langle \;\middle|\; \text{R23: } \dots \right\rangle$

$= \mathrm{PA}\left\langle \;\middle|\; \begin{matrix}\text{VR123:}\\ \text{R23}\end{matrix} \dots ;\ \text{D: } \dots \right\rangle$

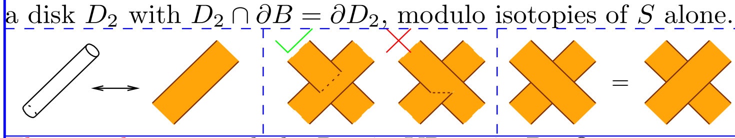
{w-Tangles} = v-Tangles / OC: …



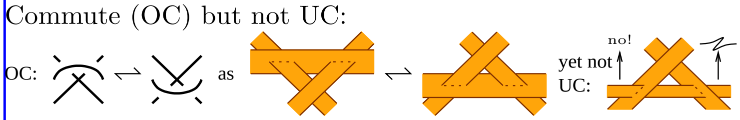
The w−generators.

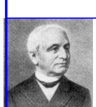


A Ribbon 2-Knot is a surface S embedded in $\mathbb{R}^4$ that bounds an immersed handlebody B, with only "ribbon singularities"; a ribbon singularity is a disk D of trasverse double points, whose preimages in B are a disk D_1 in the interior of B and a disk D_2 with $D_2 \cap \partial B = \partial D_2$, modulo isotopies of S alone.



The w-relations include R234, VR1234, D, Overcrossings Commute (OC) but not UC:

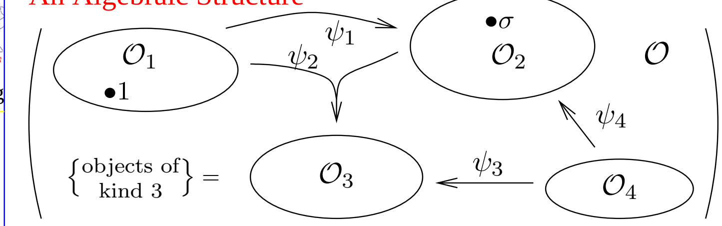




"God created the knots, all else in topology is the work of mortals."
Leopold Kronecker (modified)

Also see http://www.math.toronto.edu/~drorbn/papers/WKO/

"An Algebraic Structure"



- Has kinds, objects, operations, and maybe constants.
- Perhaps subject to some axioms.
- We always allow formal linear combinations.

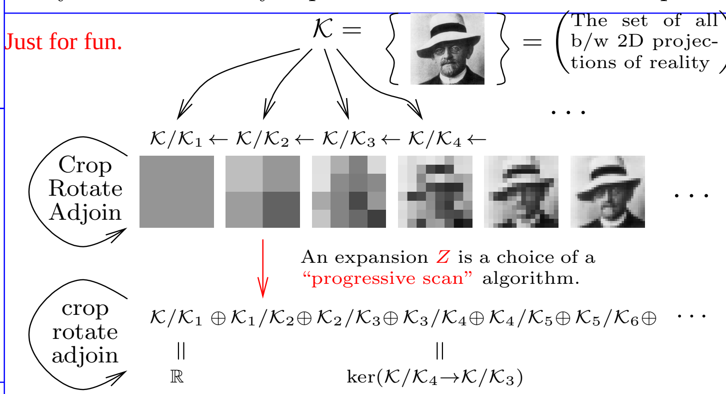
Homomorphic expansions for a filtered algebraic structure $\mathcal{K}$:

$$\mathrm{ops} \circlearrowleft \mathcal{K} = \mathcal{K}_0 \supset \mathcal{K}_1 \supset \mathcal{K}_2 \supset \mathcal{K}_3 \supset \dots$$
$$\Downarrow \qquad\qquad \downarrow Z$$
$$\mathrm{ops} \circlearrowleft \mathrm{gr}\,\mathcal{K} := \mathcal{K}_0/\mathcal{K}_1 \oplus \mathcal{K}_1/\mathcal{K}_2 \oplus \mathcal{K}_2/\mathcal{K}_3 \oplus \mathcal{K}_3/\mathcal{K}_4 \oplus \dots$$

An expansion is a filtered $Z : \mathcal{K} \to \mathrm{gr}\,\mathcal{K}$ that "covers" the identity on $\mathrm{gr}\,\mathcal{K}$. A homomorphic expansion is an expansion that respects all relevant "extra" operations.

Reality. $\mathrm{gr}\,\mathcal{K}$ is often too hard. An $\mathcal{A}$-expansion is a graded "guess" $\mathcal{A}$ with a surjection $\tau : \mathcal{A} \to \mathrm{gr}\,\mathcal{K}$ and a filtered $Z : \mathcal{K} \to \mathcal{A}$ for which $(\mathrm{gr}\,Z)\circ\tau = I_{\mathcal{A}}$. An $\mathcal{A}$-expansion confirms $\mathcal{A}$ and yields an ordinary expansion. Same for "homomorphic".

Just for fun.



Filtered algebraic structures are cheap and plenty. In any $\mathcal{K}$, allow formal linear combinations, let $\mathcal{K}_1 = \mathcal{I}$ be the ideal generated by differences (the "augmentation ideal"), and let $\mathcal{K}_m := \langle(\mathcal{K}_1)^m\rangle$ (using all available "products"). In this case, set $\mathrm{proj}\,\mathcal{K} := \mathrm{gr}\,\mathcal{K}$.

Examples. 1. The projectivization of a group is a graded associative algebra.

2. Pure braids — PB_n is generated by x_{ij}, "strand i goes around strand j once", modulo "Reidemeister moves". $A_n := \mathrm{gr}\,PB_n$ is generated by $t_{ij} := x_{ij} - 1$, modulo the $4T$ relations $[t_{ij}, t_{ik} + t_{jk}] = 0$ (and some lesser ones too). Much happens in A_n, including the Drinfel'd theory of associators.

3. Quandle: a set Q with an op $\wedge$ s.t.

$$1 \wedge x = 1, \quad x \wedge 1 = x, \qquad \text{(appetizers)}$$
$$(x \wedge y) \wedge z = (x \wedge z) \wedge (y \wedge z). \qquad \text{(main)}$$

$\mathrm{proj}\,Q$ is a graded Leibniz algebra: Roughly, set $\bar{v} := (v - 1)$ (these generate $I!$), feed $1 + \bar{x}$, $1 + \bar{y}$, $1 + \bar{z}$ in (main), collect the surviving terms of lowest degree:

$$(\bar{x} \wedge \bar{y}) \wedge \bar{z} = (\bar{x} \wedge \bar{z}) \wedge \bar{y} + \bar{x} \wedge (\bar{y} \wedge \bar{z}).$$

Kashiwara, Vergne, Alekseev, Enriquez, Torossian.

Video and more at http://www.math.toronto.edu/~drorbn/Talks/Montpellier-1006/