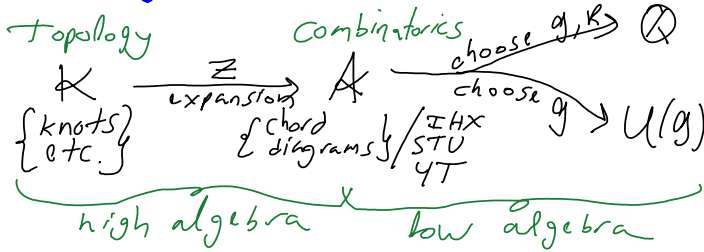
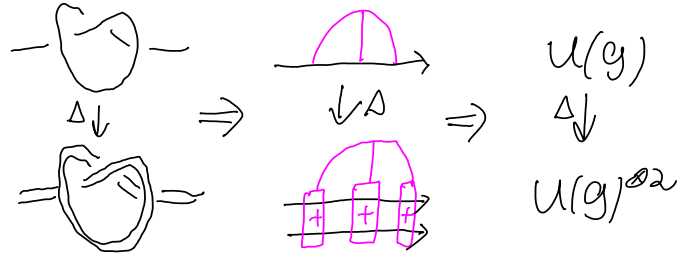


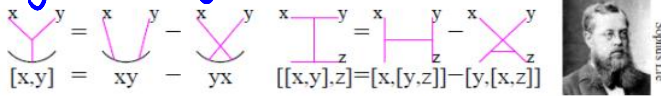
The big picture, "u" case.



What's Δ ?



very low algebra.



More precisely, let $\mathfrak{g} = \langle X_a \rangle$ be a Lie algebra with an orthonormal basis, and let $R = \langle v_\alpha \rangle$ be a representation.

Set $f_{abc} := \langle [a,b], c \rangle$ and then $X_a v_\beta = \sum_{\gamma} r_{a\gamma}^\beta v_\gamma$

$W_{\mathfrak{g}, R} : \text{Diagram} \rightarrow \sum_{abc\alpha\beta\gamma} f_{abc} r_{a\gamma}^\beta r_{b\alpha}^\gamma r_{c\beta}^\alpha$

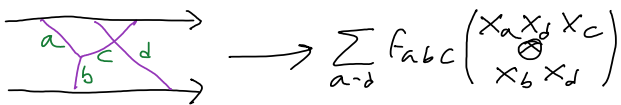
Exercise. Find a fast method to find $W_{\mathfrak{g}, R}(D)$ when $\mathfrak{g} = \mathfrak{gl}_n$, $R = \mathbb{R}^n$. Is it related to the Conway polynomial?

Universal Representation Theory.

Inspired by $p([x,y]) = p(x)p(y) - p(y)p(x)$, set $U(\mathfrak{g}) = \langle \text{words in } \mathfrak{g} \rangle / [x,y] = xy - yx$.
 * Every rep of \mathfrak{g} extends to $U(\mathfrak{g})$.
 * $\exists \Delta: U(\mathfrak{g}) \rightarrow U(\mathfrak{g})^{\otimes 2}$ by "word splitting", as must be for $R \otimes R$.

Exercise. With $\mathfrak{g} = \langle x, y \rangle / [x,y] = x$, determine $U(\mathfrak{g})$. Guess a generalization.

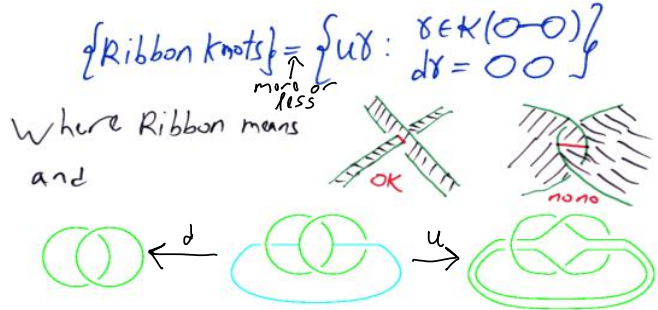
Low algebra. $A(\uparrow) \rightarrow U(\mathfrak{g})^{\otimes 2}$ via



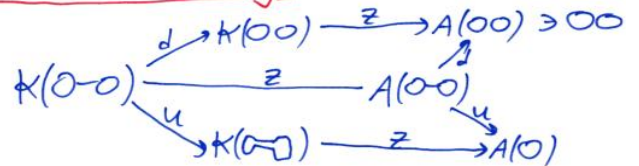
& likewise, $A(\uparrow_n) \rightarrow U(\mathfrak{g})^{\otimes n} \Rightarrow$

$A(\uparrow_n)$ is "universal universal rep. theory"!

A "Homomorphic Expansion" $Z: \mathcal{K} \rightarrow \mathcal{A}$ is an expansion that intertwines all relevant algebraic ops. If \mathcal{K} is finitely presented, finding Z is High Algebra.



Algebraic knot theory:

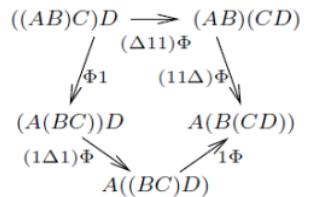


So $Z(\{\text{Ribbon knots}\}) \subset \{u\alpha : \alpha = Z(O-O)\} \subset A(O-O)$

$\forall \alpha \left[\begin{array}{|c} \square \\ \hline \square \end{array} \right] = 0$, follows from $\begin{array}{|c} \cup \\ \hline \cup \end{array} = \begin{array}{|c} \cup \\ \hline \cup \end{array}$

An Associator: Quantum Algebra's "root object"

$(AB)C \xrightarrow{\Phi \in U(\mathfrak{g})^{\otimes 3}} A(BC)$



satisfying the "pentagon",

$\Phi \cdot (1\Delta 1) \Phi \cdot 1\Phi = (\Delta 1 1) \Phi \cdot (11\Delta) \Phi$

The hexagon? Never heard of it.



See Also. B-N & Dancso, arXiv: 1103.1896