

After $A \mapsto A/\sqrt{k}$, and setting $\hbar = \frac{1}{\sqrt{k}}$:

$$Z(\gamma) = \int_{A \in \mathcal{L}(\mathbb{R}^3, \mathfrak{g})} \mathcal{D}A \operatorname{tr}_R \operatorname{hol}_\gamma(A) e^{\frac{i}{4\pi} \int_{\mathbb{R}^3} \operatorname{tr} (A \wedge dA + \frac{2}{3} A \wedge A \wedge A)} e^{CS(A)}$$

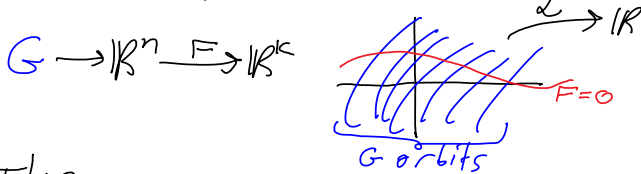
where $\operatorname{tr}_R \operatorname{hol}_\gamma(A) = \operatorname{tr}_R (1 + \hbar \int ds A(\dot{\gamma}(s)))$

Trouble? "d" is not invertible!
 $+ \hbar^2 \int_{s_1 < s_2} A(\dot{\gamma}(s_1)) A(\dot{\gamma}(s_2)) + \dots$

Gauge Invariance: $CS(A)$ is invariant under $A \mapsto A + dA$, $dA = -(dC + \hbar[A, C])$, $C \in \mathcal{L}^0(\mathbb{R}^3, \mathfrak{g})$

Back to the drawing board....

Suppose $\mathcal{L}(x)$ on \mathbb{R}^n is invariant under a k -dimensional group G w/ Lie algebra $\mathfrak{g} = \langle \mathfrak{g}_a \rangle$, and suppose $F: \mathbb{R}^n \rightarrow \mathbb{R}^k$ is such that $F=0$ is a section of the G -action:



Then

$$\int_{\mathbb{R}^n} dx e^{i\mathcal{L}} \sim \int_{\mathbb{R}^n} dx e^{i\mathcal{L}} \delta(F(x)) \cdot \det \left(\frac{\partial F_a}{\partial g_b} \right) (x)$$

$$\sim \int_{\mathbb{R}^n} dx \int_{\mathbb{R}^k} d\phi e^{i(\mathcal{L} + F(x) \cdot \phi)} \det \left(\frac{\partial F_a}{\partial g_b} \right) (x)$$

} Perturbation theory for determinants?

$$\det(J_0 + \hbar J_1(x)) = \det(J_0) \sum_m \hbar^m \operatorname{Tr} (\Lambda^m J_0^{-1}) \cdot (\Lambda^m J_1(x))$$

Berezin Fermionic Anti-commuting Variables: $\int d^k \bar{c} d^k c e^{i\bar{c} J_0^{-1} c} \sim \det(J_0)$

So $Z \sim \int_{\mathbb{R}^n} dx \int_{\mathbb{R}^k} d\phi \int d^k \bar{c} \int d^k c e^{i\mathcal{L}_{tot}}$ where

$$\mathcal{L}_{tot} = \underbrace{\mathcal{L}(x)}_{\text{the original}} + \underbrace{F(x) \cdot \phi}_{\text{gauge-fixing}} + \underbrace{\bar{c} \left(\frac{\partial F_a}{\partial g_b} \right) c}_{"ghosts"}$$

In Chern-Simons, w/ $F(A) := d^*A = \partial_i A^i$, get

$$\mathcal{L}_{tot} = \frac{k}{4\pi} \int_{\mathbb{R}^3} \operatorname{tr} (A \wedge dA + \frac{2}{3} A \wedge A \wedge A + \partial_i \bar{c} A^i + \bar{c} \partial_i (\partial^i + ad A^i) c)$$

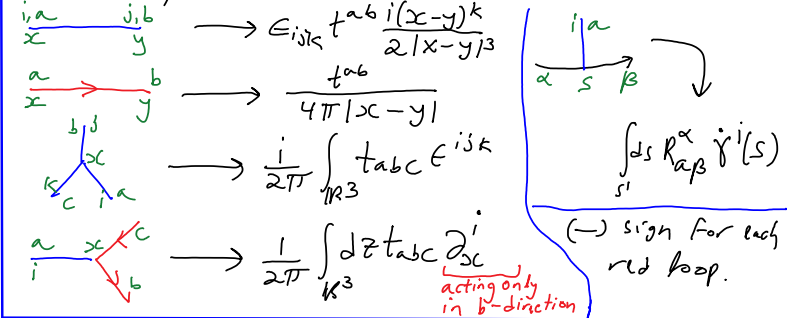
So we have

- * A bosonic quadratic term involving $(\frac{A}{\partial})$.
- * A fermionic quadratic term involving \bar{c}, c .
- * A cubic interaction of 3 A's.
- * A cubic $A\bar{c}c$ vertex.
- * Funny A and γ "holonomy" vertices along γ .

After much crunching:

$$Z(\gamma) = \sum_{m=0}^{\infty} \hbar^m \sum_{\text{Feynman diags } D} \mathcal{E}(D) \mathcal{O} =$$

where $\mathcal{E}(D)$ is constructed as follows:



By a bit of a miracle, this boils down to a configuration space integral, which in itself can be reduced to a pre-image count.
 ... But I run out of steam for tonight...



Banks like knots.

"God created the knots, all else in topology is the work of mortals."
 Leopold Kronecker (modified)



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