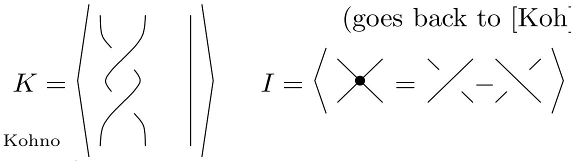
Dror Bar−Natan and Peter Lee in Oregon, August 2011

http://www.math.toronto.edu/~drorbn/Talks/Oregon-1108/

Example. (goes back to [Koh])


T. Kohno



$(K/I^{p+1})^\star = (\text{invariants of type } p) =: \mathcal{V}_p$

$(I^p/I^{p+1})^\star = \mathcal{V}_p/\mathcal{V}_{p-1}$ $\quad V = \langle t^{ij} | t^{ij} = t^{ji}\rangle = \langle | \ \vdash\!\dashv \rangle$

$\ker \bar\mu_2 = \langle [t^{ij}, t^{kl}] = 0 = [t^{ij}, t^{ik} + t^{jk}]\rangle = \langle \text{4T relations}\rangle$

$A = q(K) = \left(\text{horizontal chord diagrams mod 4T}\right) = \langle \ldots \rangle / 4T$

Z: universal finite type invariant, the Kontsevich integral.

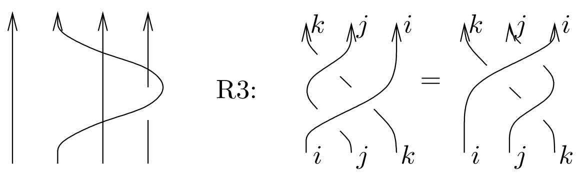
PvB_n is the group


L. Kauffman [Kau, KL]

$$\langle \sigma_{ij} : 1 \le i \ne j \le n\rangle \Big/ \begin{matrix}\sigma_{ij}\sigma_{ik}\sigma_{jk} = \sigma_{jk}\sigma_{ik}\sigma_{ij} \\ \sigma_{ij}\sigma_{kl} = \sigma_{kl}\sigma_{ij}\end{matrix}$$

of "pure virtual braids" ("braids when you look", "blunder braids"):

$\sigma_{24} =$ R3:

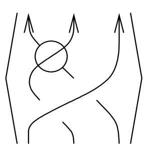


The Main Theorem [Lee]. PvB_n is quadratic.

$A_n = q(PvB_n)$. [GPV]


Goussarov-Polyak-Viro

$I =$



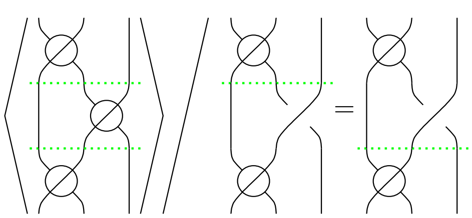
with $\tilde\sigma_{ij} = \sigma_{ij} - 1$, the "semi-virtual crossing".

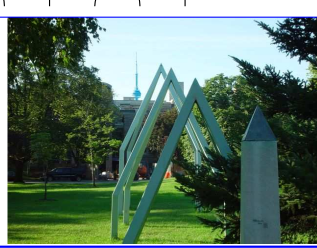
$V = I/I^2 = \left\langle \text{v-braids with one semi-virtual crossing}\right\rangle / (\text{virtual} = \text{real}) = \langle a_{ij}\rangle_{1\le i\ne j\le n}$ $\quad a_{24} =$

$A_n = TV/\langle [a_{ij}, a_{ik}] + [a_{ij}, a_{jk}] + [a_{ik}, a_{jk}],\ c^{ij}_{kl} = [a_{ij}, a_{kl}]\rangle$,

$y_{ijk} = \ldots$

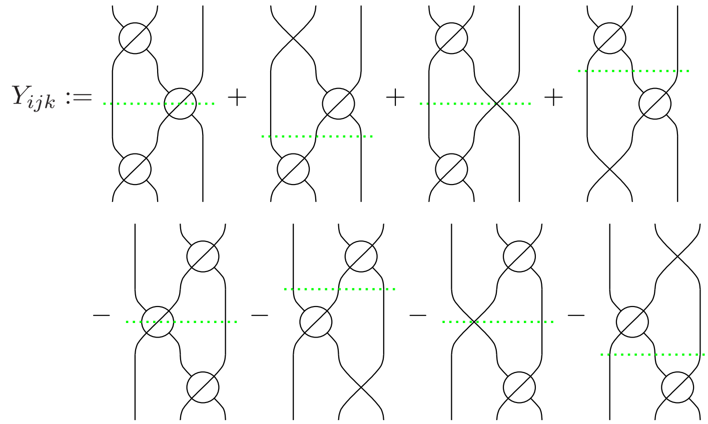
$I^{:p}$.





James Gillespie's Sightline #2 (1984) is a syzygy, and (arguably) Toronto's largest sculpture. Find it next to University of Toronto's Hart House.

$\mathfrak{R}_2(PvB_n)$ is generated as a vector space by C^{ij}_{kl} and



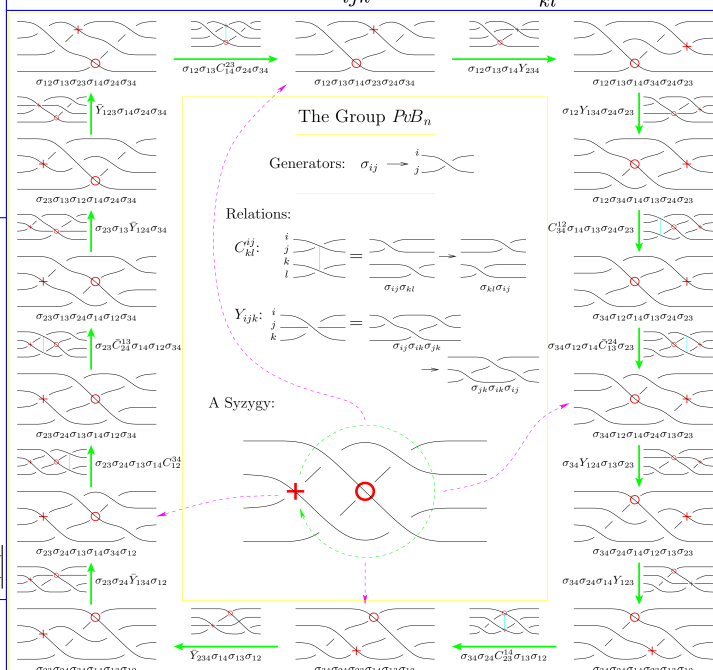
Syzygy Completeness, for PvB_n, means:

$$\mathfrak{R}_p = \bigoplus_{j=1}^{p-1}\mathfrak{R}_{p,j} \xrightarrow{\partial} I^{:p} \xrightarrow{\pi} V^{\otimes p}$$

$\{\tilde\sigma_{12} : \underline{Y_{345}} : \tilde\sigma_{67} : \ldots\} \longrightarrow$

$\{\tilde\sigma_{12} : Y_{345} : \tilde\sigma_{67} : \ldots\} \longrightarrow \{a_{12}y_{345}a_{67}\ldots\}$

Is every relation between the y_{ijk}'s and the c^{ij}_{kl}'s also a relation between the Y_{ijk}'s and the C^{ij}_{kl}'s?



Theorem S. Let D be the free associative algebra generated by symbols a_{ij}, y_{ijk} and c^{ij}_{kl}, where $1 \le i,j,k,l \le n$ are distinct integers. Let D_0 be the part of D with only a_{ij} symbols and let D_1 be the span of the monomials in D having only a_{ij} symbols, with exactly one exception that may be either a y_{ijk} or a c^{ij}_{kl}. Let $\partial : D_1 \to D_0$ be the map defined by

$$y_{ijk} \mapsto [a_{ij}, a_{ik}] + [a_{ij}, a_{jk}] + [a_{ik}, a_{jk}],$$
$$c^{ij}_{kl} \mapsto [a_{ij}, a_{kl}].$$

Then $\ker \partial$ is generated by a family of elements readable from the picture above and by a few similar but lesser families.

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