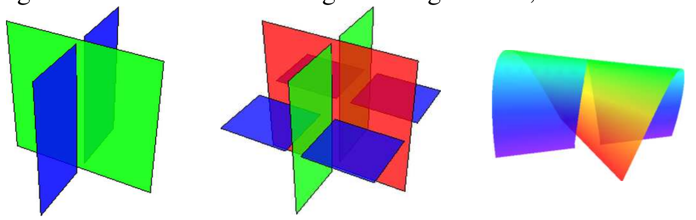
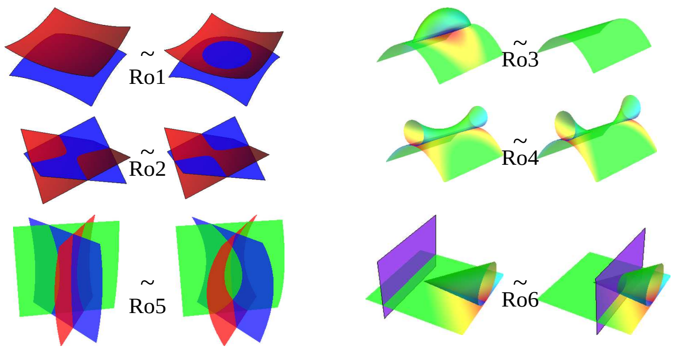
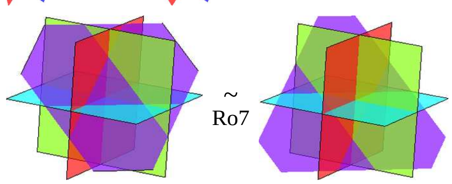
Knots in Three and Four Dimensions, 2

Theorem. Every 2-knot can be represented by a "broken surface diagram" made of the following basic ingredients,



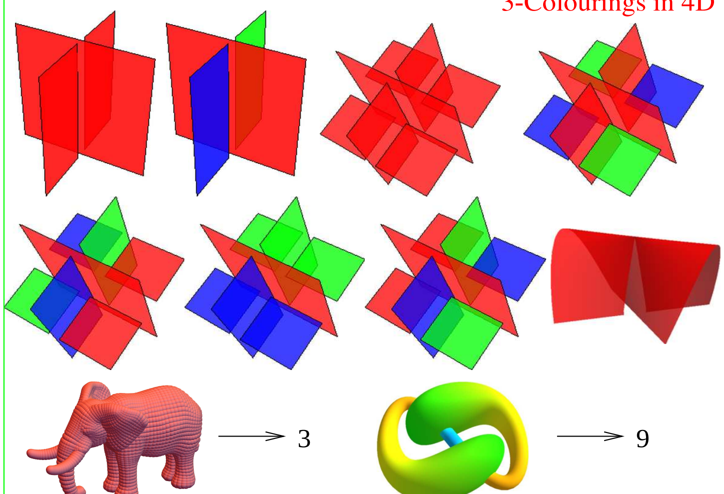
. . . and any two representations of the same knot differ by a sequence of the following "Roseman moves":





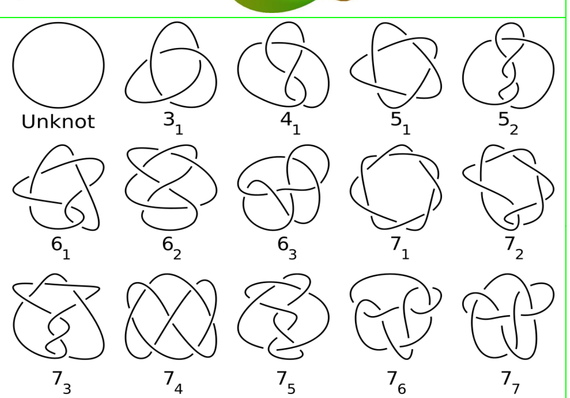

D. Roseman

3-Colourings in 4D



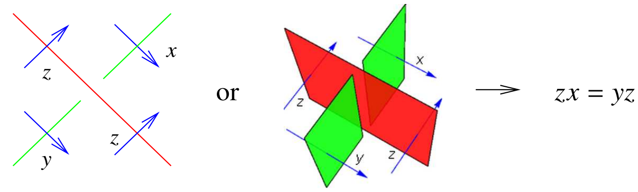
A Knot Table

There are many more!



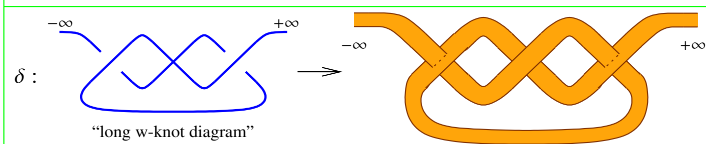
ω/KT

A Stronger Invariant. There is an assigment of groups to knots / 2-knots as follows. Put an arrow "under" every un-broken curve / surface in a broken curve / surface diagram and label it with the name of a group generator. Then mod out by relations as below.



Facts. The resulting "Fundamental group" $\pi_1(K)$ of a knot / 2-knot K is a very strong but not very computable invariant of K. Though it has computable projections; e.g., for any finite G, count the homomorphisms from $\pi_1(K)$ to G.

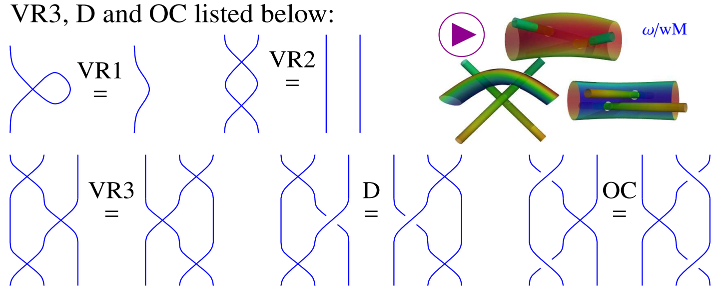
Exercise. Show that $|\mathrm{Hom}(\pi_1(K) \to S_3)| = \lambda(K) + 3$.



→ "simple long knotted 2D tube in 4D"

Satoh's Conjecture. (Satoh, *Virtual Knot Presentations of Ribbon Torus-Knots,* J. Knot Theory and its Ramifications **9** (2000) 531–542). Two long w-knot diagrams represent via the map δ the same simple long 2D knotted tube in 4D iff they differ by a sequence of R-moves as above and the "w-moves" VR1–VR3, D and OC listed below:


Shin Satoh



Some knot theory books.

- Colin C. Adams, *The Knot Book, an Elementary Introduction to the Mathematical Theory of Knots,* American Mathematical Society, 2004.
- Meike Akveld and Andrew Jobbings, *Knots Unravelled, from Strings to Mathematics,* Arbelos 2011.
- J. Scott Carter and Masahico Saito, *Knotted Surfaces and Their Diagrams,* American Mathematical Society, 1997.
- Peter Cromwell, *Knots and Links,* Cambridge University Press, 2004.
- W.B. Raymond Lickorish, *An Introduction to Knot Theory,* Springer 1997.