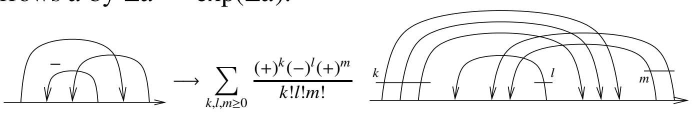
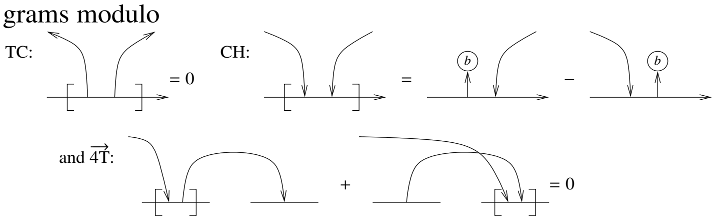
Warning. Conventions on this page change randomly from line to line.

$Z^{w/2}$. The GGA story is about $Z^{w/2}\colon \mathcal{K}\to\mathcal{A}^{w/2}$, defined on arrows a by $\pm a\mapsto\exp(\pm a)$:



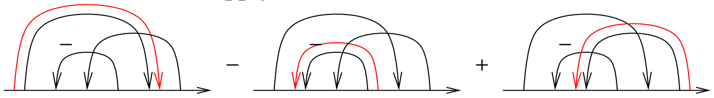
Where the target space $\mathcal{A}^{w/2}$ is the space of unsigned arrow diagrams modulo



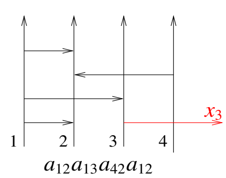
($Z^{w/2}$ is a reduction of the much-studied Z^w [BND, BN]).

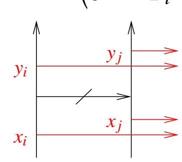
The Euler Trick. How best do non-commutative algebra with exponentials? Logarithms are from hell as $e^fe^g=e^{\mathrm{bch}(f,g)}$, but Euler's from heaven: Let E be the derivation $Ef:=(\deg f)f$ $(=xf'$, in $\mathbb{Q}[\![x]\!])$ and let $\tilde{E}Z:=Z^{-1}EZ$ $(=x(\log Z)'$ in same). If $\deg x=1$ then $\tilde{E}e^x=x$ and if $F=e^f$ and $G=e^g$, then $\tilde{E}(FG)$ is
$(FG)^{-1}((EF)G+F(EG))=G^{-1}(\tilde{E}F)G+\tilde{E}G=e^{-\operatorname{ad}g}(\tilde{E}F)+\tilde{E}G.$

Scatter and Glow. Apply $\tilde{E}$ to $Z(K)$. EZ is shown:



Tail scattering. The algebra $\mathbb{Q}[\![b_i]\!]\langle a_{ij}\rangle$ modulo $[a_{ij},a_{kl}]=0$ (loc), $[a_{ij},a_{ik}]=0$ (TC), and $[a_{ik},a_{jk}]=-[a_{ij},a_{jk}]=b_ja_{ik}-b_ia_{jk}$ (CH and $\overrightarrow{4T}$), acts on $V=\mathbb{Q}[\![b_i]\!]\langle x_i=a_{i\infty}\rangle$ by $[a_{ij},x_i]=0$, $[a_{ij},x_j]=b_ix_j-b_jx_i$. Hence $e^{\operatorname{ad}a_{ij}}x_i=x_i$, $e^{\operatorname{ad}a_{ij}}x_j=e^{b_i}x_j+\frac{b_j}{b_i}(1-e^{b_i})x_i$. Renaming $\bar{x}_i=x_i/b_i$, $T_i=e^{b_i}$, get $[e^{\operatorname{ad}a_{ij}}]_{\bar{x}_i,\bar{x}_j}=\begin{pmatrix}1 & 1-T_i\\ 0 & T_i\end{pmatrix}$. Alternatively,



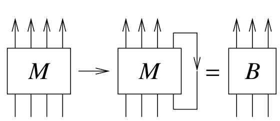


$\longrightarrow\quad \bar{y}_i=\bar{x}_i$

$\bar{y}_j=(1-T_i)\bar{x}_i+T_i\bar{x}_j$

Linear Control Theory.

If $\begin{pmatrix}y\\ y_n\end{pmatrix}=\begin{pmatrix}\Xi & \phi\\ \theta & \alpha\end{pmatrix}\begin{pmatrix}x\\ x_n\end{pmatrix}$, and we further impose $x_n=y_n$, then $y=Bx$ where $B=\Xi+\dfrac{\phi\theta}{1-\alpha}$. This fully explains the Gassner formulas and the GGA formula!



All that remains now is to replace TC by something more interesting: with $\epsilon^2=0$,

$$[a_{ij},a_{ik}]=\epsilon(c_ja_{ik}-c_ka_{ij}).$$

Many further changes are also necessary, and the algebra is a lot more complicated and revolves around "quantization of Lie bialgebras" [EK, En]. But the spirit is right.

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Video and more at http://www.math.toronto.edu/~drorbn/Talks/NCSU-1604/