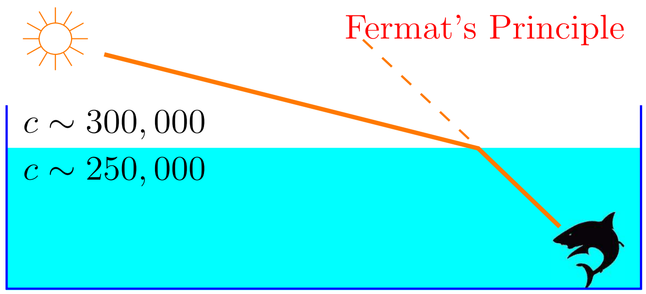
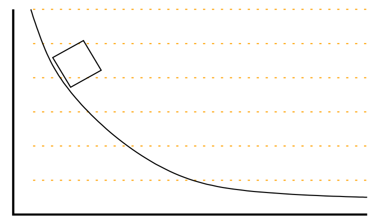
The Hardest Math I've Ever Really Used, 2

Picture credits. Mona: Leonrado; Al Gore: Futurama; Map 1: en.wikipedia.org/wiki/Greenhouse_gas; Smokestacks: gbuapcd.org/complaint.htm; Penguin: brentpabst.com/bp/2007/12/15/BrentGoesPenguin.aspx; Map 2: flightpedia.org; Segway: co2calculator.wordpress.com/2008/10; Lobachevsky: en.wikipedia.org/wiki/Nikolai_Lobachevsky; Eschers: www.josleys.com/show_gallery.php?galid=325;

Fermat's Principle



The Brachistochrone



0, $\sqrt{10}$, $\sqrt{20}$, $\sqrt{30}$, $\sqrt{40}$, $\sqrt{50}$

$$mgh = \tfrac{1}{2}mv^2$$

Bernoulli on Newton. "I recognize the lion by his paw".

Flatlanders airline route map

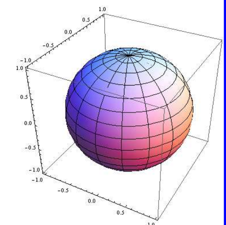
576
252
167
131
112
103
100
103
112

The Least Action Principle.

Everywhere in physics, a system goes from A to B along the path of least action.

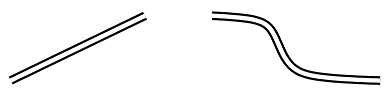
With small print for quantum mechanics.

```
ParametricPlot3D[{
  Sin[u] Cos[v],
  Sin[u] Sin[v],
  Cos[u]
 }, {u, 0, π}, {v, 0, 2π}]
```

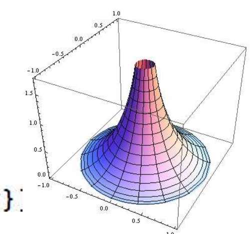


The Happy Segway Principle

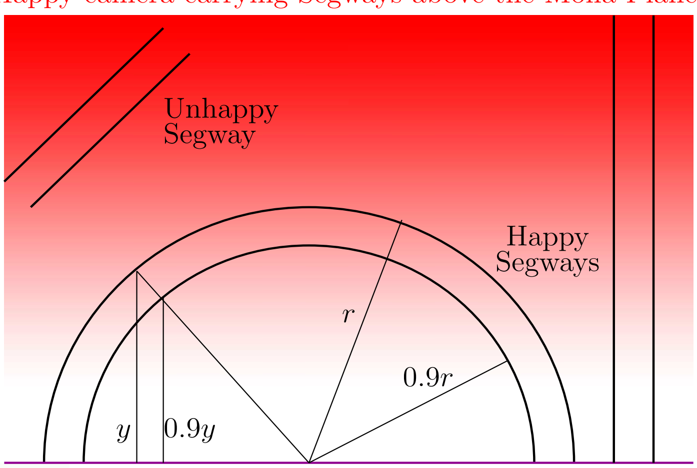
A Segway is happy iff both its wheels are



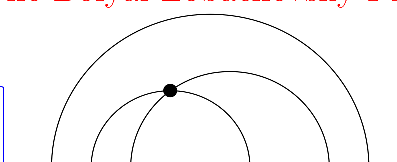

```
ParametricPlot3D[{
  Sech[u] Cos[v],
  Sech[u] Sin[v],
  u - Tanh[u]
 }, {u, 0, e}, {v, 0, 2π}]
```



Happy camera-carrying Segways above the Mona Plane

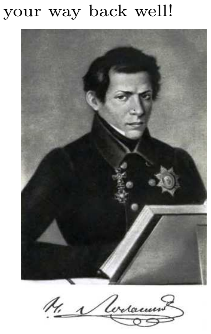


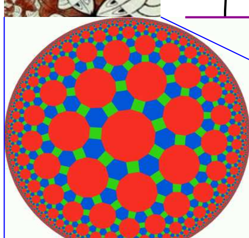
The Bolyai-Lobachevsky Plane



Two parallels through one point

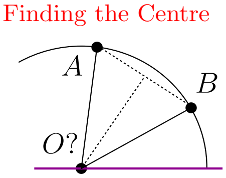
Further Fun Facts. • In small scale, $\pi^H \to \pi^E$. In large scale, $\pi^H \to \infty$. • The sum of the angles of a triangle is always less than π. In fact, sum+area= π, so the largest possible area of a triangle is π. • If your friend walks away, she'll drop out of sight before you know it. • There are so many places just a stone throw away! But you'd better remember your way back well!



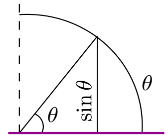


Some further basic geometry also occurs:

Finding the Centre



Parametrization



$$\theta'(t) = \sin\theta(t)$$

$$\Downarrow$$

$$\theta = 2\arctan e^t$$

The Actual Code

```
p3.y = p2.y + b*x3p;
x = p1.x-p2.x; y = p1.y-p2.y;
d1 = p1.d; d2 = p2.d;
norm = sqrt(x*x + y*y);
a = x/norm; b = y/norm;
x1p = a*x + b*y;
x0 = (x1p + (d1*d1-d2*d2)/x1p)/2;
r = sqrt((x1p-x0)*(x1p-x0)+d1*d1);
x1pp = (x1p-x0)/r; x2pp = -x0/r;
theta1 = acos(x1pp);
theta2 = acos(x2pp);
t1 = log(tan(theta1/2));
t2 = log(tan(theta2/2));
t3 = t1 + s*(t2-t1);
theta3 = 2*atan(exp(t3));
x3pp = cos(theta3);
d3pp = sin(theta3);
x3p = x0 + r*x3pp;
p3.d = r*d3pp;
p3.x = p2.x + a*x3p;
```

Ops used. $+$, $-$, $\times$, $\div$, $\sqrt{\ }$, cos, sin, tan, arccos, arctan, log, exp.

Video at http://www.math.toronto.edu/~drorbn/Talks/RCI-110213/, more at http://www.math.toronto.edu/~drorbn/Talks/Niagara-1612/