Demo Programs for 0-Co. ωεβ/Demo

The R-matrices

```
R⁺0,i_,j_ := 𝔼[bᵢ cⱼ + bᵢ⁻¹ (e^bᵢ − 1) uᵢ wⱼ];
R⁻0,i_,j_ := 𝔼[−bᵢ cⱼ + bᵢ⁻¹ (e^−bᵢ − 1) uᵢ wⱼ];
```

Utilities

```
CF[ω_. 𝔼[Q_]] := Simplify[ω] 𝔼[Simplify[Q]];
𝔼 /: 𝔼[Q1_] 𝔼[Q2_] := CF@𝔼[Q1 + Q2];
ω1_. 𝔼[Q1_] ≡ ω2_. 𝔼[Q2_] := Simplify[ω1 == ω2 ∧ Q1 == Q2];
```

Normal Ordering Operators

```
N(x:w|u)i_ cj_→k_[ω_. 𝔼[Q_]] := CF[
   ω 𝔼[e^γ α xₖ + γ cₖ + (Q /. cⱼ | xᵢ → 0)] /. {γ → ∂_cⱼ Q, α → ∂_xᵢ Q}];
Nwi_ uj_→k_[ω_. 𝔼[Q_]] := CF[
   ν ω 𝔼[−bₖ ν α β + ν β uₖ + ν α wₖ + ν δ uₖ wₖ + (Q /. wᵢ | uⱼ → 0)] /.
     ν → (1 + bₖ δ)⁻¹ /.
    {α → ∂_wᵢ Q /. uⱼ → 0, β → ∂_uⱼ Q /. wᵢ → 0, δ → ∂_wᵢ,uⱼ Q}];
```

Stitching

```
mi_,j_→k_[Z_] := Module[{x, z},
  CF[(Z // Nwᵢ uⱼ→x // Ncᵢ uₓ→x // Nwₓ cⱼ→x) /. z_i|j|x → zₖ]]
```

Some calculations for T_0

```
T0 = R⁺0,5,1 R⁺0,2,4 R⁻0,3,6
```

$$\mathbb{E}\left[b_5 c_1 + b_2 c_4 - b_3 c_6 + \frac{(-1+e^{b5})\,u_5 w_1}{b_5} + \frac{(-1+e^{b2})\,u_2 w_4}{b_2} + \frac{(-1+e^{-b3})\,u_3 w_6}{b_3}\right]$$

```
T0 // m1,2→1 // m3,4→3 // m3,5→3 // m3,6→3
```

$$\frac{1}{1-(-1+e^{b1})(-1+e^{b3})}\,\mathbb{E}\Big[b_3 c_1 + b_1 c_3 - b_3 c_3 + \frac{e^{b3}(-1+e^{b1})(-1+e^{b3})\,u_1 w_1}{(-e^{b1}-e^{b3}+e^{b1+b3})\,b_1} - \frac{e^{b1}(-1+e^{b3})\,u_3 w_1}{(-1+(-1+e^{b1})(-1+e^{b3}))\,b_3} - \frac{e^{-b3}(-1+e^{b3})\,u_3 w_3}{b_3} - \frac{e^{-b3}(-1+e^{b1})(-e^{b3} b_3 u_1 + e^{b1}(-1+e^{b3})\,b_1 u_3)\,w_3}{b_1 (b_3 - (-1+e^{b1})(-1+e^{b3})\,b_3)}\Big]$$

Verifying meta-associativity

```
Q0 = 𝔼[Sum[fᵢ cᵢ, {i, 3}] + Sum[fᵢ,ⱼ uᵢ wⱼ, {i, 3}, {j, 3}]]
```

$\mathbb{E}[c_1 f_1 + c_2 f_2 + c_3 f_3 + u_1 w_1 f_{1,1} + u_1 w_2 f_{1,2} + u_1 w_3 f_{1,3} + u_2 w_1 f_{2,1} + u_2 w_2 f_{2,2} + u_2 w_3 f_{2,3} + u_3 w_1 f_{3,1} + u_3 w_2 f_{3,2} + u_3 w_3 f_{3,3}]$

```
(Q0 // m1,2→1 // m1,3→1) ≡ (Q0 // m2,3→2 // m1,2→1)
True
```

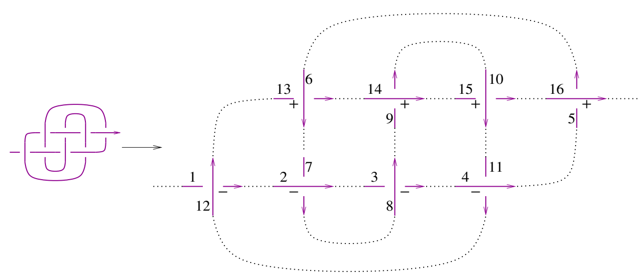
Testing R3

```
t1 = R⁺0,1,2 R⁺0,3,4 R⁺0,5,6 // m3,5→x // m1,6→y // m2,4→z
```

$$\mathbb{E}\left[b_x c_y + b_x c_z + b_y c_z + \frac{e^{bx}(-1+e^{by})\,u_y w_z}{b_y} + \frac{(-1+e^{bx})\,u_x (w_y+w_z)}{b_x}\right]$$

```
t1 ≡ (R⁺0,1,2 R⁺0,3,4 R⁺0,5,6 // m1,3→x // m2,5→y // m4,6→z)
True
```

8_{17}




```
z1 = R⁻0,12,1 R⁻0,2,7 R⁻0,8,3 R⁻0,4,11 R⁺0,16,5 R⁺0,6,13 R⁺0,14,9 R⁺0,10,15;
Do[z1 = (z1 // m1,n→1) /. b_ → b, {n, 2, 16}];
{CF@z1, KnotData[{8, 17}, "AlexanderPolynomial"][t]}
```

$$\left\{-\frac{e^{3b}\,\mathbb{E}[0]}{1-4e^{b}+8e^{2b}-11e^{3b}+8e^{4b}-4e^{5b}+e^{6b}},\ 11-\frac{1}{t^3}+\frac{4}{t^2}-\frac{8}{t}-8t+4t^2-t^3\right\}$$

Demo Programs for 1-Co. ωεβ/Demo

```
Λ[k_] := ((tₖ − 1) (2 (α β + δ μ)² − α² β²) − 4 vₖ cₖ wₖ δ² μ² −
     δ (1 + μ) (wₖ² α² + vₖ² β²) − vₖ² wₖ² δ³ (1 + 3 μ) −
     2 (α β + 2 δ μ + vₖ wₖ δ² (1 + 2 μ) + 2 cₖ δ μ²) (wₖ α + vₖ β) −
     4 (cₖ μ² + vₖ wₖ δ (1 + μ)) (α β + δ μ)) (1 + tₖ) / 4;
```

The Λόγος

The Generators

```
R⁺i_,j_ := 𝔼[1, Log[tᵢ] cⱼ, vᵢ wⱼ, vᵢ cᵢ wⱼ + cᵢ cⱼ + vᵢ² wⱼ²/4];
R⁻i_,j_ := 𝔼[1, −Log[tᵢ] cⱼ, −tᵢ⁻¹ vᵢ wⱼ,
    tᵢ⁻¹ vᵢ cⱼ wⱼ − cᵢ cⱼ − tᵢ⁻² vᵢ² wⱼ²/4];
(uri_ := 𝔼[tᵢ^−1/2, 0, 0, cᵢ tᵢ⁻²]; nri_ := 𝔼[tᵢ^1/2, 0, 0, −cᵢ tᵢ²];)
```

Differential Polynomials

```
DPx_→Dα_,y_→Dβ_[P_][f_] := (* means P[∂α,∂β][f] *)
  Total[CoefficientRules[P, {x, y}] /.
    ({m_, n_} → c_) :→ c D[f, {α, m}, {β, n}]]
```

Utilities

```
CF[ℰ_𝔼] := Expand /@ Together /@ ℰ;
𝔼 /: 𝔼[ω1_, L1_, Q1_, P1_] 𝔼[ω2_, L2_, Q2_, P2_] :=
  CF@𝔼[ω1 ω2, L1 + L2, ω2 Q1 + ω1 Q2, ω2⁴ P1 + ω1⁴ P2];
```

Normal Ordering Operators

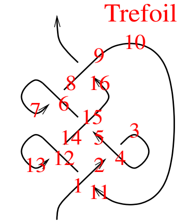
```
Ncj_ (x:v|w)i_→k_[𝔼[ω_, L_, Q_, P_]] := With[{q = e^γ β xₖ + γ cₖ}, CF[
    𝔼[ω, γ cₖ + (L /. cⱼ → 0), ω e^γ β xₖ + (Q /. xᵢ → 0),
      e^−q DPcⱼ→Dγ,xᵢ→Dβ[P][e^q]] /. {γ → ∂_cⱼ L, β → ω⁻¹ ∂_xᵢ Q}]];
Nwi_ vj_→k_[𝔼[ω_, L_, Q_, P_]] :=
  With[{q = ((1 − tₖ) α β + β vₖ + α wₖ + δ vₖ wₖ) / μ}, CF[
    𝔼[μ ω, L, μ ω q + μ (Q /. wᵢ | vⱼ → 0),
      μ⁴ e^−q DPwᵢ→Dα,vⱼ→Dβ[P][e^q] + ω⁴ Λ[k]] /. μ → 1 + (tₖ − 1) δ /.
    {α → ω⁻¹ (∂_wᵢ Q /. vⱼ → 0), β → ω⁻¹ (∂_vⱼ Q /. wᵢ → 0),
     δ → ω⁻¹ ∂_wᵢ,vⱼ Q}]];
```

Stitching

```
mi_,j_→k_[Z_𝔼] := Module[{x, z},
  CF[(Z // Nwᵢ vⱼ→x // Ncᵢ vₓ→x // Nwₓ cⱼ→x) /. z_i|j|x → zₖ]]
```

The 0-Framed Trefoil

```
z2 = R⁺1,11 R⁻4,2 nr3 R⁺15,5 R⁻6,8 ur7 R⁺9,16 nr10 R⁻12,14 ur13;
(Do[z2 = z2 // m1,k→1, {k, 2, 16}];
 z2 = z2 /. a_1 :→ a)
```



$$\mathbb{E}\Big[-1+\frac{1}{t}+t,\ 0,\ 0,\ 16+\frac{2c}{t^4}-\frac{1}{t^3}-\frac{6c}{t^3}+\frac{4}{t^2}+\frac{10c}{t^2}-\frac{10}{t}-\frac{8c}{t}-18t+8ct+14t^2-10ct^2-7t^3+6ct^3+2t^4-2ct^4+2vw-\frac{2vw}{t^4}+\frac{4vw}{t^3}-\frac{6vw}{t^2}+\frac{2vw}{t}-6tvw+4t^2vw-2t^3vw\Big]$$

Questions and To Do List. • Clean up and write up. • Implement well, compute for everything in sight. • Why are our quantities polynomials rather than just rational functions? • Bounds on their degrees? • Their integrality ($\mathbb{Z}$) properties? • Can everything be re-stated using integrals ($\int$)? • Find the 2-variable version (for knots). How complex is it? • What about links / closed components? • Fully digest the "expansion" theorem; include cuaps. • Explore the (non-)dependence on R. • Is there a canonical R? • What does "group like" mean? • Strand removal? Strand doubling? Strand reversal? • Say something about knot genus. • Find the EK/AT/KV "vertex". • Use as a playground to study associators/braidors. • Restate in topological language. • Study the associated (v-)braid representations. • Study mirror images and the $\mathfrak{b}^+ \leftrightarrow \mathfrak{b}^-$ involution. • Study ribbon knots. • Make precise the relationship with Γ-calculus and Alexander. • Relate to the coloured Jones polynomial. • Relate with "ordinary" q-algebra. • k-smidgen sl_n, etc. • Are there "solvable" CYBE algebras not arising from semi-simple algebras? • Categorify and appease the Gods.

This is http://www.math.toronto.edu/~drorbn/Talks/MIT-1612/. Better videos at .../Indiana-1611/, .../LesDiablerets-1608/