```
Define[op_is__ = ℰ_] :=
 Module[{SD, ii, jj, kk, isp, nis, nisp, sis},
  Block[{i, j, k},
   ReleaseHold[Hold[
      SD[op_{nisp,$k_Integer}, Block[{i, j, k}, op_{isp,$k} = ℰ;
         op_{nis,$k}]];
      SD[op_{isp}, op_{{is},$k}]; SD[op_{sis__}, op_{{sis}}];
     ] /. {SD → SetDelayed,
      isp → {is} /. {i → i_, j → j_, k → k_},
      nis → {is} /. {i → ii, j → jj, k → kk},
      nisp → {is} /. {i → ii_, j → jj_, k → kk_}
     }] ]]
```

The Fundamental Tensors

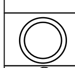
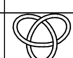
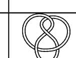
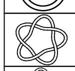
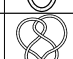
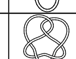
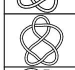
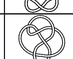
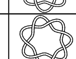
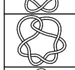
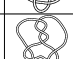
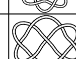
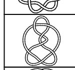
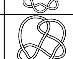
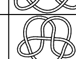
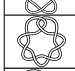
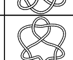
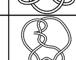
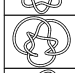
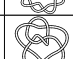
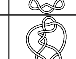
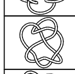
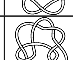
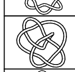
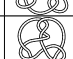
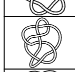
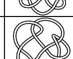
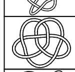
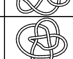
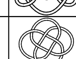
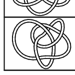
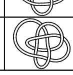
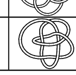
```
Define[am_{i,j→k} = 𝔼_{{i,j}→{k}}[(α_i + α_j) a_k, (e^{-γ α_j} ξ_i + ξ_j) x_k, 1]_{$k},
 bm_{i,j→k} = 𝔼_{{i,j}→{k}}[(β_i + β_j) b_k, (η_i + η_j) y_k, e^{(e^{-ϵ β_i} - 1) η_j y_k}]_{$k}]
Define[R_{i,j} =
  𝔼_{{}→{i,j}}[ħ a_j b_i, ħ x_j y_i, e^( Sum_{k=2}^{$k+1} (1 - e^{γ ϵ ħ})^k (ħ y_i x_j)^k / (k (1 - e^{k γ ϵ ħ})) )]_{$k}]
Define[R̄_{i,j} = 𝔼_{{}→{i,j}}[-ħ a_j b_i, -ħ x_j y_i / B_i,
   1 + If[$k == 0, 0, (R̄_{{i,j},$k-1})_{$k}[3] -
      (((R̄_{{i,j},0})_{$k} R_{1,2} (R̄_{{3,4},$k-1})_{$k}) // (bm_{i,1→i} am_{j,2→j}) //
         (bm_{i,3→i} am_{j,4→j}))[3]]],
 P_{i,j} = 𝔼_{{i,j}→{}}[β_i α_j / ħ, η_i ξ_j / ħ,
   1 + If[$k == 0, 0, (P_{{i,j},$k-1})_{$k}[3] -
      (R_{1,2} // ((P_{{1,j},0})_{$k} (P_{{i,2},$k-1})_{$k}))[3]]]]
Define[aS_j = R̄_{i,j} ~ B_i ~ P_{i,j},
 aS̄_i = 𝔼_{{i}→{i}}[-a_i α_i, -x_i 𝒜_i ξ_i,
   1 + If[$k == 0, 0, (aS̄_{{i},$k-1})_{$k}[3] -
      ((aS̄_{{i},0})_{$k} ~ B_i ~ aS_i ~ B_i ~ (aS̄_{{i},$k-1})_{$k})[3]]]]
Define[bS_i = R_{i,1} ~ B_1 ~ aS_1 ~ B_1 ~ P_{i,1},
 bS̄_i = R_{i,1} ~ B_1 ~ aS̄_1 ~ B_1 ~ P_{i,1},
 aΔ_{i→j,k} = (R_{1,j} R_{2,k}) // bm_{1,2→3} // P_{3,i},
 bΔ_{i→j,k} = (R_{j,1} R_{k,2}) // am_{1,2→3} // P_{i,3}]
```

```
Define[
 dm_{i,j→k} =
  (𝔼_{{i,j}→{i,j}}[β_i b_i + α_j a_j, η_i y_i + ξ_j x_j, 1]
      (aΔ_{i→1,2} // aΔ_{2→2,3} // aS̄_3) (bΔ_{j→-1,-2} // bΔ_{-2→-2,-3})) //
   (P_{-1,3} P_{-3,1} am_{2,j→k} bm_{i,-2→k}),
 dS_i = 𝔼_{{i}→{1,2}}[β_i b_1 + α_i a_2, η_i y_1 + ξ_i x_2, 1] // (bS̄_1 aS_2) //
   dm_{2,1→i},
 dΔ_{i→j,k} = (bΔ_{i→3,1} aΔ_{i→2,4}) // (dm_{3,4→k} dm_{1,2→j})]
Define[C_i = 𝔼_{{}→{i}}[0, 0, B_i^{1/2} e^{-ħ ϵ a_i/2}]_{$k},
 C̄_i = 𝔼_{{}→{i}}[0, 0, B_i^{-1/2} e^{ħ ϵ a_i/2}]_{$k},
 Kink_i  = (R_{1,3} C̄_2) // dm_{1,2→1} // dm_{1,3→i},
 K̄ink_i  = (R̄_{1,3} C_2) // dm_{1,2→1} // dm_{1,3→i}]
Define[
 b2t_i = 𝔼_{{i}→{i}}[α_i a_i - β_i t_i / γ, ξ_i x_i + η_i y_i, e^{ϵ β_i a_i / γ}]_{$k},
 t2b_i = 𝔼_{{i}→{i}}[α_i a_i - τ_i γ b_i, ξ_i x_i + η_i y_i, e^{ϵ τ_i a_i}]_{$k}]
Define[kR_{i,j} = R_{i,j} // (b2t_i b2t_j) /. t_{i|j} → t,
 kR̄_{i,j} = R̄_{i,j} // (b2t_i b2t_j) /. {t_{i|j} → t, T_{i|j} → T},
 km_{i,j→k} = (t2b_i t2b_j) // dm_{i,j→k} //
   b2t_k /. {t_k → t, T_k → T, τ_{i|j} → 0},
 kC_i = C_i // b2t_i /. T_i → T, kC̄_i = C̄_i // b2t_i /. T_i → T,
 kKink_i = Kink_i // b2t_i /. {t_i → t, T_i → T},
 kK̄ink_i = K̄ink_i // b2t_i /. {t_i → t, T_i → T}]
```

The Trefoil

```
$k = 2; Z = kR_{1,5} kR_{6,2} kR_{3,7} kC̄_4 kK̄ink_8 kK̄ink_9 kK̄ink_10;
Do[Z = Z ~ B_{1,r} ~ km_{1,r→1}, {r, 2, 10}];
Simplify /@ Z /. ν_1 :> ν
```

$$\mathbb{E}_{\{\}\to\{1\}}\Big[0,\ 0,\ \frac{T}{1-T+T^2}+\frac{1}{(1-T+T^2)^3}T\hbar\left(2a\left(-1+T-T^3+T^4\right)+T\left(-1+2T-3T^2+2T^3\right)\gamma-2\left(1+T^3\right)xy\gamma\hbar\right)\epsilon+\frac{1}{2\left(1-T+T^2\right)^5}T\hbar^2\Big(4a^2\left(1-T+T^2\right)^2\left(1+T-6T^2+T^3+T^4\right)+4a\left(1-T+T^2\right)\gamma\left(T\left(2-5T+8T^2-7T^3-2T^4+2T^5\right)-2\left(-1-2T+5T^2-4T^3+T^4+2T^5\right)xy\hbar\right)+\gamma^2\left(T\left(1-2T+4T^2-2T^3+6T^5-11T^6+4T^7\right)+4\left(-1+2T+T^3+T^4+2T^6-T^7\right)xy\hbar+6\left(1-T+T^2\right)^2\left(1+3T+T^2\right)x^2y^2\hbar^2\right)\Big)\epsilon^2+O[\epsilon]^3\Big]$$

diagram	n_k^t / Today's ρ_1^+	Alexander's ω^+	genus / ribbon unknotting # / amphi?	diagram	n_k^t / Today's ρ_1^+	Alexander's ω^+	genus / ribbon unknotting # / amphi?	diagram	n_k^t / Today's ρ_1^+	Alexander's ω^+	genus / ribbon unknotting # / amphi?
	0_1^a 0	1	0 / ✔ 0 / ✔		3_1^a t	$t-1$	1 / ✘ 1 / ✘		4_1^a 0	$3-t$	1 / ✘ 1 / ✔
	5_1^a $2t^3+3t$	t^2-t+1	2 / ✘ 2 / ✘		5_2^a $5t-4$	$2t-3$	1 / ✘ 1 / ✘		6_1^a $t-4$	$5-2t$	1 / ✔ 1 / ✘
	6_2^a t^3-4t^2+4t-4	$-t^2+3t-3$	2 / ✘ 1 / ✘		6_3^a 0	t^2-3t+5	2 / ✘ 1 / ✔		7_1^a $3t^5+5t^3+6t$	t^3-t^2+t-1	3 / ✘ 3 / ✘
	7_2^a $14t-16$	$3t-5$	1 / ✘ 1 / ✘		7_3^a $-9t^3+8t^2-16t+12$	$2t^2-3t+3$	2 / ✘ 2 / ✘		7_4^a $32-24t$	$4t-7$	1 / ✘ 2 / ✘
	7_5^a $9t^3-16t^2+29t-28$	$2t^2-4t+5$	2 / ✘ 2 / ✘		7_6^a $t^3-8t^2+19t-20$	$-t^2+5t-7$	2 / ✘ 1 / ✘		7_7^a $8-3t$	t^2-5t+9	2 / ✘ 1 / ✘
	8_1^a $5t-16$	$7-3t$	1 / ✘ 1 / ✘		8_2^a $2t^5-8t^4+10t^3-12t^2+13t-12$	$-t^3+3t^2-3t+3$	3 / ✘ 2 / ✘		8_3^a 0	$9-4t$	1 / ✘ 2 / ✔
	8_4^a $3t^3-8t^2+6t-4$	$-2t^2+5t-5$	2 / ✘ 2 / ✘		8_5^a $-2t^5+8t^4-13t^3+20t^2-22t+24$	$-t^3+3t^2-4t+5$	3 / ✘ 2 / ✘		8_6^a $5t^3-20t^2+28t-32$	$-2t^2+6t-7$	2 / ✘ 2 / ✘
	8_7^a $-t^5+4t^4-10t^3+12t^2-13t+12$	t^3-3t^2+5t-5	3 / ✘ 1 / ✘		8_8^a $-t^3+4t^2-12t+16$	$2t^2-6t+9$	2 / ✔ 2 / ✘		8_9^a 0	$-t^3+3t^2-5t+7$	3 / ✔ 1 / ✔
	8_{10}^a $-t^5+4t^4-11t^3+16t^2-21t+20$	t^3-3t^2+6t-7	3 / ✘ 2 / ✘		8_{11}^a $5t^3-24t^2+39t-44$	$-2t^2+7t-9$	2 / ✘ 1 / ✘		8_{12}^a 0	$t^2-7t+13$	2 / ✘ 2 / ✔
	8_{13}^a $-t^3+4t^2-14t+20$	$2t^2-7t+11$	2 / ✘ 1 / ✘		8_{14}^a $5t^3-28t^2+57t-68$	$-2t^2+8t-11$	2 / ✘ 1 / ✘		8_{15}^a $21t^3-64t^2+120t-140$	$3t^2-8t+11$	2 / ✘ 2 / ✘
	8_{16}^a $t^5-6t^4+17t^3-28t^2+35t-36$	t^3-4t^2+8t-9	3 / ✘ 2 / ✘		8_{17}^a 0	$-t^3+4t^2-8t+11$	3 / ✘ 1 / ✔		8_{18}^a 0	$-t^3+5t^2-10t+13$	3 / ✘ 2 / ✔
	8_{19}^n $-3t^5-4t^2-3t$	t^3-t^2+1	3 / ✘ 3 / ✘		8_{20}^n $4t-4$	t^2-2t+3	2 / ✔ 1 / ✘		8_{21}^n $t^3-8t^2+16t-20$	$-t^2+4t-5$	2 / ✘ 1 / ✘

Video and more at http://www.math.toronto.edu/~drorbn/Talks/Ohio-1901