

Define[lhs = rhs, ...] defines the lhs to be rhs, except that rhs is computed only once for each value of \$k. Fancy Mathematica notation for the faint of heart. Most readers should ignore.

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SetAttributes[Define, HoldAll];
Define[def_, defs_] := (Define[def]; Define[defs]);
Define[op_is_ = \_ ] :=
Module[{SD, ii, jj, kk, isp, nis, nisp, sis},
Block[{i, j, k},
ReleaseHold[Hold[
SD[op_nisp, $k_Integer, PPBoot@Block[{i, j, k}, op_isp, $k = \_ ;
op_nis, $k];];
SD[op_isp, op_{is}, $k]; SD[op_sis_, op_{sis}];
] /. {SD -> SetDelayed,
isp -> {is} /. {i -> i_, j -> j_, k -> k_},
nis -> {is} /. {i -> ii, j -> jj, k -> kk},
nisp -> {is} /. {i -> ii_, j -> jj_, k -> kk_}
}]]]

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The Objects

$\omega\epsilon\beta$ /objects

Symmetric Algebra Objects

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sm_{i,j} -> r_{k} :=
E_{\{i,j\} -> \{k\}} [b_k (\beta_i + \beta_j) + t_k (\tau_i + \tau_j) + a_k (\alpha_i + \alpha_j) +
y_k (\eta_i + \eta_j) + x_k (\xi_i + \xi_j)];
s\Delta_{i,j} -> r_{k} :=
E_{\{i\} -> \{j,k\}} [\beta_i (b_j + b_k) + \tau_i (t_j + t_k) + \alpha_i (a_j + a_k) +
\eta_i (y_j + y_k) + \xi_i (x_j + x_k)];
sS_{i,j} := E_{\{i\} -> \{i\}} [-\beta_i b_i - \tau_i t_i - \alpha_i a_i - \eta_i y_i - \xi_i x_i];
se_{i,j} := E_{\{\} -> \{i\}} [\theta];
s\eta_{i,j} := E_{\{i\} -> \{\}} [\theta];
s\sigma_{i,j} := E_{\{i\} -> \{j\}} [\beta_i b_j + \tau_i t_j + \alpha_i a_j + \eta_i y_j + \xi_i x_j];
sY_{i,j} -> r_{k,l,m} := E_{\{i\} -> \{j,k,l,m\}} [\beta_i b_k + \tau_i t_k + \alpha_i a_l + \eta_i y_j + \xi_i x_m];

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The CU Definitions

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c\Lambda = \left( \eta_i + \frac{e^{-\gamma \alpha_i - \epsilon \beta_i} \eta_j}{1 + \gamma \epsilon \eta_j \xi_i} \right) y_k + \left( \beta_i + \beta_j + \frac{\text{Log}[1 + \gamma \epsilon \eta_j \xi_i]}{\epsilon} \right) b_k +
\left( \alpha_i + \alpha_j + \frac{\text{Log}[1 + \gamma \epsilon \eta_j \xi_i]}{\gamma} \right) a_k + \left( \frac{e^{-\gamma \alpha_j - \epsilon \beta_j} \xi_i}{1 + \gamma \epsilon \eta_j \xi_i} + \xi_j \right) x_k;
Define[cm_{i,j} -> k = E_{\{i,j\} -> \{k\}} [c\Lambda]];
Define[cs_{i,j} = s\sigma_{i,j} /. \tau_i -> \theta, ce_i = se_i, c\eta_i = s\eta_i,
c\Delta_{i,j,k} = s\Delta_{i,j,k},
cs_i = sS_i // sY_{i-1,2,3,4} // cm_{4,3-i} // cm_{i,2-i} // cm_{i,1-i}];

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Booting Up QU

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Define[as_{i,j} = E_{\{i\} -> \{j\}} [a_j \alpha_i + x_j \xi_i],
bs_{i,j} = E_{\{i\} -> \{j\}} [b_j \beta_i + y_j \eta_i]];
Define[am_{i,j} -> k = E_{\{i,j\} -> \{k\}} [(\alpha_i + \alpha_j) a_k + (\mathcal{A}_j^{-1} \xi_i + \xi_j) x_k],
bm_{i,j} -> k = E_{\{i,j\} -> \{k\}} [(\beta_i + \beta_j) b_k + (\eta_i + e^{-\epsilon \beta_i} \eta_j) y_k]];
Define[R_{i,j} = E_{\{\} -> \{i,j\}} [\hbar a_j b_i + \sum_{k=1}^{\$k+1} \frac{(1 - e^{\gamma \epsilon \hbar})^k (\hbar y_i x_j)^k}{k (1 - e^{k \gamma \epsilon \hbar})}],
\bar{R}_{i,j} = CF@E_{\{\} -> \{i,j\}} [-\hbar a_j b_i, -\hbar x_j y_i / B_i,
1 + If[ \$k == \theta, \theta, (\bar{R}_{\{i,j\}, \$k-1} \$k [3] -
((\bar{R}_{\{i,j\}, \theta} \$k R_{1,2} (\bar{R}_{\{3,4\}, \$k-1} \$k) // (bm_{i,1-i} am_{j,2-j}) //
(bm_{i,3-i} am_{j,4-j})) [3] ]],
P_{i,j} = E_{\{i,j\} -> \{\}} [\beta_i \alpha_j / \hbar, \eta_i \xi_j / \hbar,
1 + If[ \$k == \theta, \theta, (P_{\{i,j\}, \$k-1} \$k [3] -
(R_{1,2} // ((P_{\{1,3\}, \theta} \$k (P_{\{1,2\}, \$k-1} \$k)) [3] ])]];

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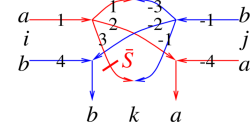
Define[as_i = (a\sigma_{i-2} \bar{R}_{1,i}) // P_{1,2},
\bar{as}_i = E_{\{i\} -> \{i\}} [-a_i \alpha_i, -x_i \mathcal{A}_i \xi_i,
1 + If[ \$k == \theta, \theta, (\bar{as}_{\{i\}, \$k-1} \$k [3] -
((\bar{as}_{\{i\}, \theta} \$k // as_i // (\bar{as}_{\{i\}, \$k-1} \$k) [3] ])]];

```

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Define[bs_i = b\sigma_{i-1} R_{i,2} // as_2 // P_{1,2},
\bar{bs}_i = b\sigma_{i-1} R_{i,2} // \bar{as}_2 // P_{1,2},
a\Delta_{i-j,k} = (R_{1,j} R_{2,k}) // bm_{1,2-3} // P_{3,1},
b\Delta_{i-j,k} = (R_{j,1} R_{k,2}) // am_{1,2-3} // P_{i,3}];

```



The Drinfel'd double:

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Define[
dm_{i,j} -> k =
((sY_{i-4,4,1,1} // a\Delta_{1-1,2} // a\Delta_{2-2,3} // \bar{as}_3)
(sY_{j-1,-1,-4,-4} // b\Delta_{-1-1,-2} // b\Delta_{-2-2,-3})) //
(P_{-1,3} P_{-3,1} am_{2,-4} km_{4,-2-k}]);

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Define[d\sigma_{i,j} = a\sigma_{i,j} b\sigma_{i,j},
de_i = se_i, d\eta_i = s\eta_i,
dS_i = sY_{i-1,1,2,2} // (\bar{bs}_1 as_2) // dm_{2,1-i},
\bar{dS}_i = sY_{i-1,1,2,2} // (bs_1 \bar{as}_2) // dm_{2,1-i},
d\Delta_{i-j,k} = (b\Delta_{i-3,1} a\Delta_{i-2,4}) // (dm_{3,4-k} dm_{1,2-j}]);

```

```

Define[C_i = E_{\{\} -> \{i\}} [\theta, \theta, B_i^{1/2} e^{-\hbar \epsilon a_i / 2}]_{\$k},
\bar{C}_i = E_{\{\} -> \{i\}} [\theta, \theta, B_i^{-1/2} e^{\hbar \epsilon a_i / 2}]_{\$k},
Kink_i = (R_{1,3} \bar{C}_2) // dm_{1,2-1} // dm_{1,3-i},
\bar{Kink}_i = (\bar{R}_{1,3} C_2) // dm_{1,2-1} // dm_{1,3-i}];

```

Note. $t = \epsilon a - \gamma b$ and $b = -t / \gamma + \epsilon a / \gamma$.

```

Define[b2t_i = E_{\{i\} -> \{i\}} [\alpha_i a_i + \beta_i (\epsilon a_i - t_i) / \gamma + \xi_i x_i + \eta_i y_i],
t2b_i = E_{\{i\} -> \{i\}} [\alpha_i a_i + \tau_i (\epsilon a_i - \gamma b_i) + \xi_i x_i + \eta_i y_i]];

```

The Knot Tensors

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Define[kR_{i,j} = R_{i,j} // (b2t_i b2t_j) /. t_{i|j} -> t,
\bar{kR}_{i,j} = \bar{R}_{i,j} // (b2t_i b2t_j) /. {t_{i|j} -> t, T_{i|j} -> T},
km_{i,j} -> k = (t2b_i t2b_j) // dm_{i,j-k} //
b2t_k /. {t_k -> t, T_k -> T, \tau_{i|j} -> \theta},
kC_i = C_i // b2t_i /. T_i -> T,
\bar{kC}_i = \bar{C}_i // b2t_i /. T_i -> T,
kKink_i = Kink_i // b2t_i /. {t_i -> t, T_i -> T},
\bar{kKink}_i = \bar{Kink}_i // b2t_i /. {t_i -> t, T_i -> T}];

```

Some of the Atoms.

$\omega\epsilon\beta$ /atoms

With $\mathcal{A}_i := e^{\alpha_i}$ and $B_i = e^{-b_i}$,

```

PP_ := Identity; \$k = 1; \hbar = \gamma = 1;
Column[
(# -> (\epsilon = ToExpression[#];
Normal@Simplify[\epsilon[[1]] + \epsilon[[2]] + Log@\epsilon[[3]]])] & @/
{"dm_{i,j} -> k", "d\Delta_{i,j,k}", "dS_i", "R_{i,j}", "P_{i,j}"}];

```