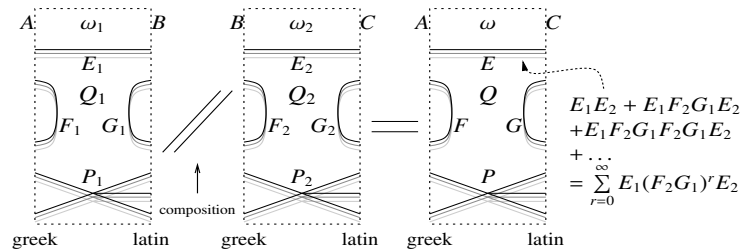


So What? If V is a representation, then $V^{\otimes n}$ explodes as a function of n , while in **DoPeGDO** up to a fixed power of ϵ , the ranks of $\text{mor}(A \rightarrow B)$ grow polynomially as a function of $|A|$ and $|B|$.

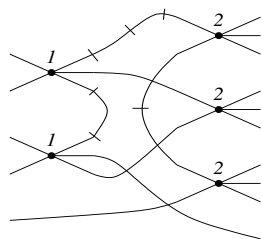
Compositions. In $\text{mor}(A \rightarrow B)$,

$$Q = \sum_{i \in A, j \in B} E_{ij} \zeta_i \zeta_j + \frac{1}{2} \sum_{i, j \in A} F_{ij} \zeta_i \zeta_j + \frac{1}{2} \sum_{i, j \in B} G_{ij} \zeta_i \zeta_j,$$

and so (remember, $e^x = 1 + x + xx/2 + xxx/6 + \dots$)



where $\bullet E = E_1(I - F_2 G_1)^{-1} E_2$.
 $\bullet F = F_1 + E_1 F_2 (I - G_1 F_2)^{-1} E_1^T$.
 $\bullet G = G_2 + E_2^T G_1 (I - F_2 G_1)^{-1} E_2$.
 $\bullet \omega = \omega_1 \omega_2 \det(I - F_2 G_1)^{-1}$.
 $\bullet P$ is computed as the solution of a messy PDE or using “connected Feynman diagrams” (yet we’re still in pure algebra!). Docility is preserved.



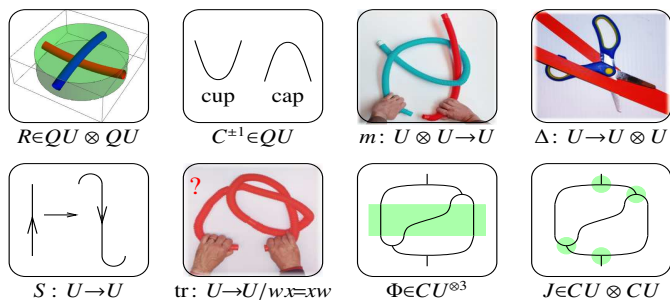
Questions. • Are there QFT precedents for “two-step Gaussian integration”?

• In QFT, one saves even more by considering “one-particle-irreducible” diagrams and “effective actions”. Does this mean anything here?

• Understanding $\text{Hom}(\mathbb{Q}[z_A] \rightarrow \mathbb{Q}[z_B])$ seems like a good cause. Can you find other applications for the technology here?

$$\left(\begin{aligned} QU &= \mathcal{U}_h(sl_{2+}^\epsilon) = A(y, b, a, x) [\hbar] \text{ with } [a, x] = x, [b, y] = -\epsilon y, [a, b] = 0, \\ [a, y] &= -y, [b, x] = \epsilon x, \text{ and } xy - qyx = (1 - AB)/\hbar, \text{ where } q = e^{\hbar \epsilon}, A = e^{-\hbar \epsilon a}, \\ \text{and } B &= e^{-\hbar b}. \text{ Also } \Delta(y, b, a, x) = (y_1 + B_1 y_2, b_1 + b_2, a_1 + a_2, x_1 + A_1 x_2), \\ S(y, b, a, x) &= (-B^{-1} y, -b, -a, -A^{-1} x), \text{ and } R = \sum \hbar^{j+k} y^j b^k \otimes a^j x^k / j! k! q^j. \end{aligned} \right)$$

Theorem. Everything of value regrading $U = CU$ and/or its quantization $U = QU$ is **DoPeGDO**:



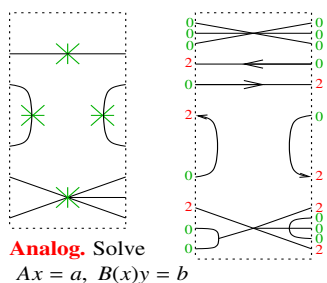
also Cartan’s θ , the Dequantizer, and more, and all of their compositions.

DoPeGDO Footnotes. Each variable has a “weight” $\in \{0, 1, 2\}$, and always $\text{wt } z_i + \text{wt } \zeta_i = 2$.

- †1. Really, “weight-graded finite sets” $A = A_0 \sqcup A_1 \sqcup A_2$.
- †2. Really, a power series in the weight-0 variables^{†5}.
- †3. The weight of Q must be 2, so it decomposes as $Q = Q_{20} + Q_{11}$. The coefficients of Q_{20} are rational numbers while the coefficients of Q_{11} may be weight-0 power series^{†5}.
- †4. Setting $\text{wt } \epsilon = -2$, the weight of P is ≤ 2 (so the powers of the weight-0 variables are not constrained)^{†5}.
- †5. In the knot-theoretic case, all weight-0 power series are rational functions of bounded degree in the exponentials of the weight-0 variables.
- †6. There’s also an obvious product $\text{mor}(A_1 \rightarrow B_1) \times \text{mor}(A_2 \rightarrow B_2) \rightarrow \text{mor}(A_1 \sqcup A_2 \rightarrow B_1 \sqcup B_2)$.

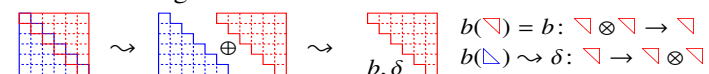
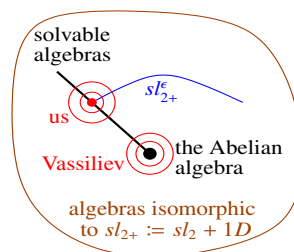
Full DoPeGDO. Compute compositions in two phases:

- A 1-1 phase over the ring of power series in the weight-0 variables, in which the weight-2 variables are spectators.
- A (slightly modified) 2-0 phase over \mathbb{Q} , in which the weight-1 variables are spectators.



Solvable Approximation. In sl_n , half is enough! Indeed $sl_n \oplus \mathfrak{a}_{n-1} = \mathcal{D}(\nabla, b, \delta)$. Now define $sl_{n+}^\epsilon := \mathcal{D}(\nabla, b, \epsilon \delta)$. Schematically, this is $[\nabla, \nabla] = \nabla$, $[\Delta, \Delta] = \epsilon \Delta$, and $[\nabla, \Delta] = \Delta + \epsilon \nabla$. The same process works for all semi-simple Lie algebras, and at $\epsilon^{k+1} = 0$ always yields a solvable Lie algebra.

4D Metrized Lie Algebras



Conclusion. There are lots of poly-time-computable well-behaved near-Alexander knot invariants: • They extend to tangles with appropriate multiplicative behaviour. • They have cabling and strand reversal formulas. $\omega \epsilon \beta / \text{akt}$

The invariant for $sl_{2+}^\epsilon / (\epsilon^2 = 0)$ (prior art: $\omega \epsilon \beta / \text{Ov}$) attains 2,883 distinct values on the 2,978 prime knots with ≤ 12 crossings. HOMFLY-PT and Khovanov homology together attain only 2,786 distinct values.

| knot diag | n_k^+ $(\rho_1^+)^+$ | Alexander’s ω^+ unknotting # / amphi? | genus / ribbon | knot diag | n_k^+ $(\rho_1^+)^+$ | Alexander’s ω^+ unknotting # / amphi? | genus / ribbon | knot diag | n_k^+ $(\rho_1^+)^+$ | Alexander’s ω^+ unknotting # / amphi? | genus / ribbon |
|-----------|---|--|----------------|-----------|--|---|----------------|-----------|---|--|----------------|
| | 0_1^+ 0 | 1 $(\rho_2^+)^+$ | 0 / ✓ 0 / ✓ | | 3_1^+ T | $T-1$ T | 1 / ✗ 1 / ✗ | | 4_1^+ 0 | $3-T$ | 1 / ✗ 1 / ✓ |
| | 5_1^+ $2T^3 + 3T$ | $T^2 - T + 1$ 0 | 2 / ✗ 2 / ✗ | | 5_2^+ $5T - 4$ | $2T - 3$ $5T - 4$ | 1 / ✗ 1 / ✗ | | 6_1^+ $T - 4$ | $5 - 2T$ | 1 / ✓ 1 / ✗ |
| | 6_2^+ $T^3 - 4T^2 + 4T - 4$ | $-T^2 + 3T - 3$ $57T^7 - 207T^6 + 557T^5 - 1207T^4 + 2177T^3 - 3387T^2 + 4507T - 510$ | 2 / ✗ 1 / ✗ | | 6_3^+ 0 | $T^2 - 3T + 5$ $-10T^4 + 120T^3 - 487T^2 + 1054T - 1362$ | 2 / ✗ 1 / ✓ | | 7_1^+ $3T^5 + 5T^3 + 6T$ | $T^3 - T^2 + T - 1$ $147^4 - 167^3 - 2937^2 + 10987 - 1598$ | 3 / ✗ 3 / ✗ |
| | $3T^8 - 217T^7 + 497T^6 + 157T^5 - 433T^4 + 1543T^3 - 3431T^2 + 5482T - 6410$ | | | | $47^8 - 337^7 + 1217^6 - 2037^5 - 1117^4 + 14997^3 - 42107^2 + 71867 - 8510$ | | | | $77^8 - 287^{10} + 777^9 - 1687^8 + 3227^7 - 5607^6 + 8917^5 - 13107^4 + 1777^3 - 22387^2 + 26047 - 2772$ | | |