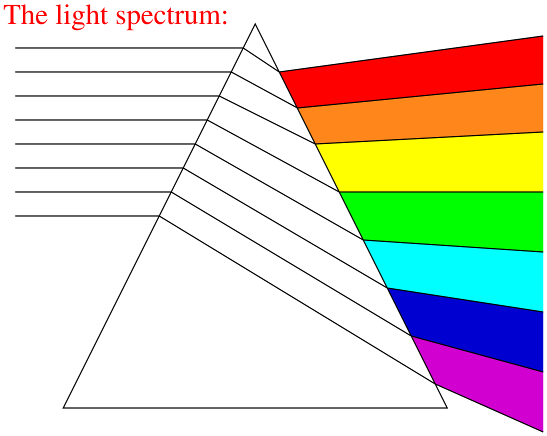
Khovanov Homology for Tangles and Cobordisms

some extra formulas and pictures

The light spectrum:



Quantum algebra:

Claim. If $ba=qab$ then

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k}_q a^k b^{n-k}$$

where

$$(n)_q := 1+q+\ldots+q^{n-1},$$

$$(n)!_q := (1)_q (2)_q \cdots (n)_q,$$

$$\binom{n}{k}_q := \frac{(n)!_q}{(k)!_q (n-k)!_q}.$$

Conjecture:

(I. Frenkel, though he may disown this version)

1. Every object in mathematics is the Euler characteristic of a complex.
2. Every operation in mathematics lifts to an operation between complexes.
3. Every identity in mathematics is true up to homotopy at complex–level.

I. Frenkel

The Jones polynomial:

Definition. $\hat{J} : \times \mapsto q)(- q^2 \asymp, \quad \hat{J} : \times \mapsto -q^{-2} \asymp + q^{-1})($,

valued in xing–free tangles mod

$\bigcirc = q + q^{-1}$

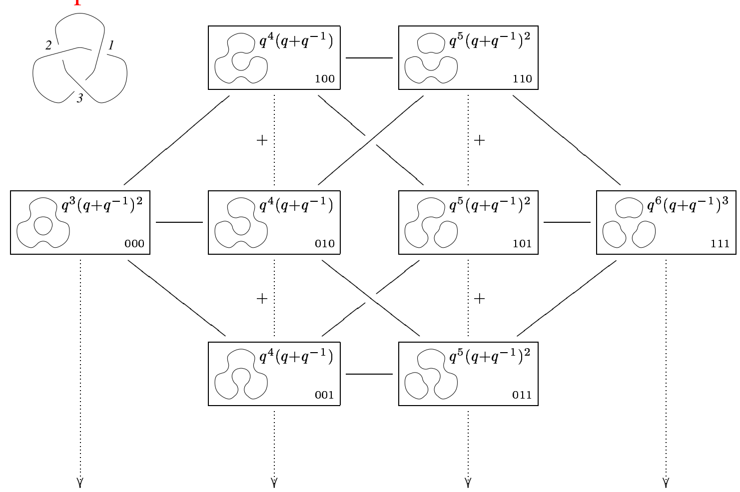
Invariance under R2:

$$\hat{J} : \;\; \mapsto \;\; -q^{-1} \; + \; + \; - q$$

$$= -q^{-1} \asymp +)(+ (q+q^{-1}) \asymp - q \asymp$$

$$=)(\qquad \square$$

Example:



$q^3(q+q^{-1})^2 \;-\; 3q^4(q+q^{-1}) \;+\; 3q^5(q+q^{-1})^2 \;-\; q^6(q+q^{-1})^3$

Complexes:

$$\Omega = \left(\Omega^{-n_-} \longrightarrow \Omega^{-n_-+1} \longrightarrow \cdots \qquad \longrightarrow \Omega^{n_+} \right)$$

Morphisms:

$$\begin{array}{ccccccccc} \cdots \longrightarrow & \Omega_0^{r-1} & \xrightarrow{d^{r-1}} & \Omega_0^{r} & \xrightarrow{d^{r}} & \Omega_0^{r+1} & \longrightarrow \cdots \\ & \downarrow F^{r-1} & & \downarrow F^{r} & & \downarrow F^{r+1} & \\ \cdots \longrightarrow & \Omega_1^{r-1} & \xrightarrow{d^{r-1}} & \Omega_1^{r} & \xrightarrow{d^{r}} & \Omega_1^{r+1} & \longrightarrow \cdots \end{array}$$

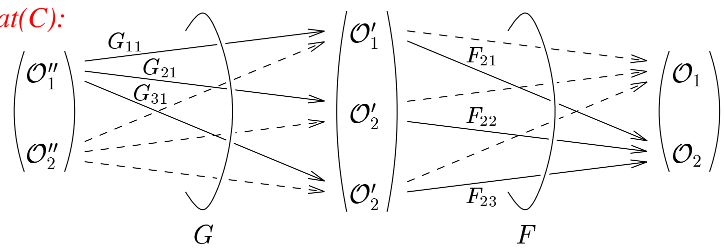
Homotopies:

$$\begin{array}{ccccc} \Omega_0^{r-1} & \xrightarrow{d^{r-1}} & \Omega_0^{r} & \xrightarrow{d^{r}} & \Omega_0^{r+1} \\ F^{r-1} \downarrow\downarrow G^{r-1} & \swarrow h^r & F^r \downarrow\downarrow G^r & \swarrow h^{r+1} & F^{r+1} \downarrow\downarrow G^{r+1} \\ \Omega_1^{r-1} & \xrightarrow{d^{r-1}} & \Omega_1^{r} & \xrightarrow{d^{r}} & \Omega_1^{r+1} \end{array}$$

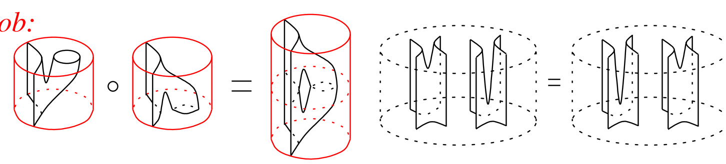
$$F^r - G^r = h^{r+1} d^r + d^{r-1} h^r$$

All arrows in an arbitrary additive category!

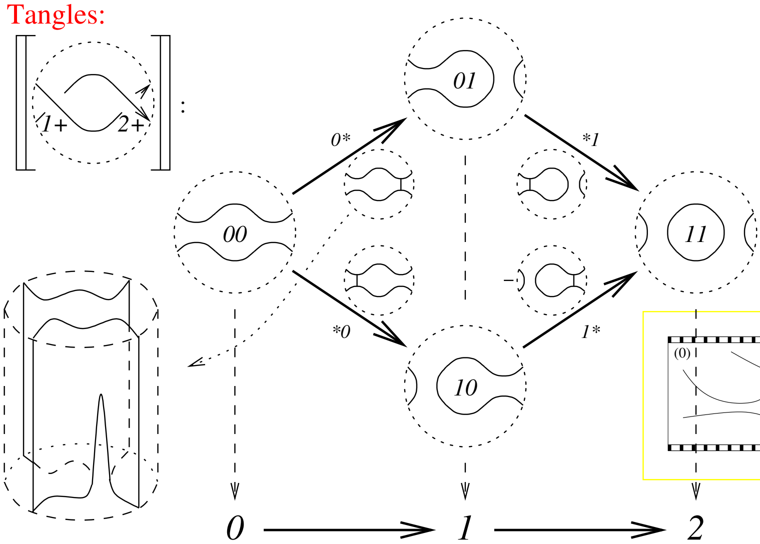
Mat(C):



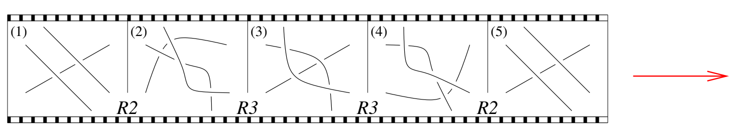
Cob:

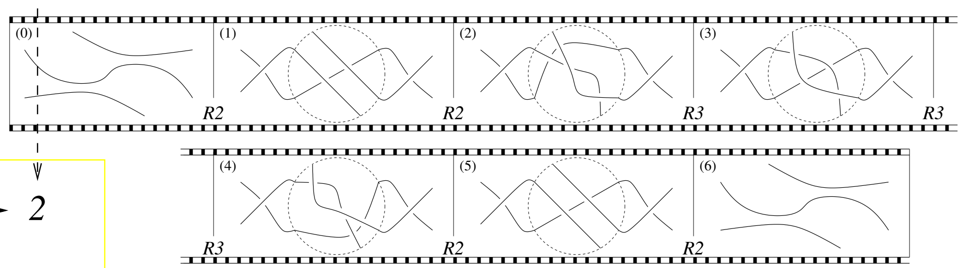


Tangles:



Movie moves:





Dror Bar–Natan, Harvard, October 2004. See www.math.toronto.edu/~drorbn/Talks/Harvard–0410/ and .../~drorbn/papers/Cobordism