Mathematics

Department of Mathematics University of Toronto September 1995

Fields Institute Comes to U. of T.

In May, 1993 a high profile international selection committee unanimously recommended that the permanent home for the Fields Institute for Research in Mathematical Science be located at the University of Toronto. The world class quality of the Mathematics Department and the commitment by the senior administration to developing mathematical sciences were cited among the major reasons for the decision.

The Fields Institute was established in 1991 with the mandate to promote leading edge research in mathematical science, to increase opportunities for graduate and postdoctoral training in Ontario and Canada and to foster interactions between university-based research and users of mathematics in the private sector. It is funded by the Province of Ontario and by NSERC, and is currently located at a temporary site at the University of Waterloo.

The Fields Institute under construction, May 1995

To house the Institute the University is constructing a new building, located on College between St. George and Huron with the Koffler Centre (old Metro Library) on one side and Architecture on the other. Construction should be completed by October 1995.

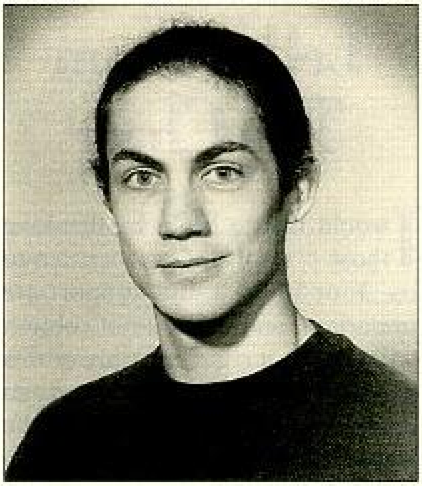
The arrival of the Institute will provide a major opportunity for the Department and the University, because it will attract a steady stream of strong international mathematical scientists in a wide variety of fields. This will provide a major resource not only for our faculty, but for our graduate students and postdoctoral fellows as well. The mandate of the Institute is the broad spectrum of mathematical science, and so its presence at the University will help us focus our efforts to develop a sense of intellectual community among the mathematical scientists in the many departments where mathematics is important.

Letter From The Chair

I would like to begin by thanking those of you who have written to us. Your interest and support are important, and our alumni column will now be a regular feature of this newsletter. You will be interested and proud to know that donations to the Department reached an all time high in 1993-95, coming from graduates and faculty alike. In particular it is a great pleasure to announce the J.R.G. Smyth Scholarship in mathematics, and a new endowment establishing the Malcolm Slingsby Robertson Fellowships for graduate students in mathematics.

This is an eventful time for mathematics and for universities, and a particularly eventful time for mathematics at U. of T. A Provostial Review has recommended that the University plan to develop into one of the major international centres for mathematical science, and the Mathematics Department is playing an important role in the Task Force charged with this responsibility. A new building to house the Fields Institute for Research in Mathematical Sciences is nearing completion on College Street between St. George and Huron, and the fiftieth anniversary meeting of the Canadian Mathematical Society has just taken place on our campus, followed by two international conferences, one in *Special Functions, q-Series, and Related Topics* and the other in *Partial Differential Equations and Their Applications*. Both were organized by members of the Department.

The Department has been extremely fortunate in appointing young tenure stream faculty of the highest calibre. With the arrival of the Fields Institute and the positive atmosphere towards mathematical science in the University, we expect to be able to continue to attract young mathematicians at the top of their fields. Our plan to play a significant role in developing mathematical science also includes closer co-operation with other departments, revision of our specialist program and recruiting top undergraduate and graduate students. In addition we are actively participating in a program with McGill, UBC and three leading Chinese universities. Altogether, mathematics at U. of T. will be even more exciting for both students and faculty.


J.P. Grossman

Our undergraduate and graduate programs continue to flourish, and in the Putnam competition our 1992-93 team ranked second and our 1993-94 and 1994-95 teams came in the top ten, with one member **J.P. Grossman**, among the top five individuals. The Department has important teaching responsibilities for students from other areas. In 1992-93, in co-operation with Botany and Zoology, **Joe Repka** pioneered an extraordinarily successful course: Biology, Models and Mathematics, for first year biology students. In 1993-94, in co-operation with the Faculty of Education, **Ed Barbeau** introduced an exciting new course for future elementary school teachers: *Concepts in Elementary Mathematics*. It is also a real pleasure to report that a number of colleagues were identified by the first year Engineering class as among their top instructors overall.

The past two years have seen a number of milestones: **John Bland**, **Mark Spivakovsky** and **Catherine Sulem** were promoted to the rank of Full Professor, and **Fiona Murnaghan** and **Luis Seco** were promoted to Associate Professor with tenure. **Kumar Murty** won a Steacie Fellowship, **Kunio Murasugi** received the Fall Prize of the Japanese Mathematical Society, **George Elliott** and **John Friedlander** were invited to speak at the International Congress (August, 1994), **George Duff** and **Donald Coxeter** were awarded honourary degrees (respectively from Dalhousie and York) and **Jim Arthur** was the Weyl Lecturer at the Institute for Advanced Study in Princeton. Our second and third Blyth lecture series were a considerable success, with **Charles Fefferman** (Fields medallist, Princeton) lecturing on *Mathematical Problems from the Elementary Quantum Mechanics of Atoms* and **Efim Zelmanov** (Fields Medallist, Chicago) lecturing on *The Burnside Problem and Profinite Groups*.

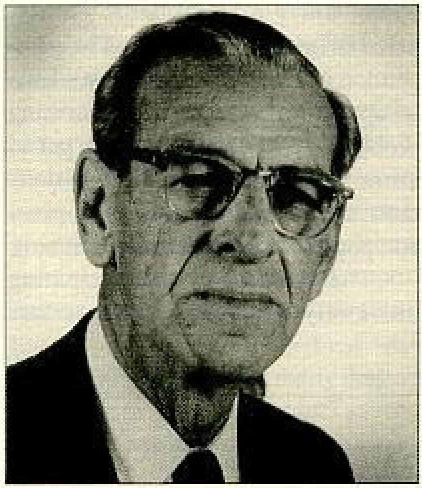
On a sadder note, I regret to report the passing of **Dan DeLury** after a long illness: Dan will be sorely missed by all of us who knew him. In his honour the Department has established the Daniel B. DeLury Teaching Award for Graduate Students in Mathematics. It is also with sadness that I report the passing of Professor **John Lighton Synge** in Blackrock, County Dublin, Ireland, at the age of 98. Synge was a Professor of Applied Mathematics in this Department for many years, and played a significant role in its development. His contribution is honoured by the Synge Prize of the Royal Society of Canada for research in mathematics. (The first two winners of the Synge Prize were **Jim Arthur** and **Michael Sigal** of this Department.)

This is the final year of my first term as Chair. **John Bland** will serve as Acting Chair next year, after which I return for an additional three years. I would like to take this opportunity to thank all the administrative and academic members of the Department for the way they have worked together to build the Department. I have also enjoyed meeting and talking with many of you, our alumni, and I hope you will continue to feel free to come back and visit.

– *Steve Halperin*

Remembering Dan DeLury

Daniel Bertrand DeLury was born 19 September, 1907 at Walker, Minnesota. His family was from Ottawa, and after primary schooling in Minnesota he went to Ottawa for secondary education, living with an uncle. He enrolled in the Mathematics, Physics and Chemistry program at University of Toronto in 1925, where another uncle, A.T. DeLury, was chairman of the Mmathematics Department, graduating in mathematics, with first class honors, in 1929 and earning his M.A. in 1930.



After that he moved for a period to University of Saskatchewan, as an instructor in Mathematics, and where the lady who would, in 1941, become his wife was a student in one of his classes. In 1934 he returned to Toronto for further graduate study obtaining his Ph.D. in 1936. His thesis was on Number Theory under the direction of Prof. W.J. Webber. It is noteworthy that another thesis some thirty years later in the Department was on a generalization of DeLury's thesis work. After a post-doctoral year at Columbia University, he was appointed a lecturer in the Department of Mathematics. Perhaps due to his long interest in biology during this period his interests changed to the area of Mathematical Statistics and he made himself an expert in this. He had risen to the rank of Assistant Professor when in 1945 he moved to Virginia Polytechnic Institute as an Associate Professor, being promoted to Professor four months later.

However, he returned to Canada in 1947 to become the Head of the Division of Statistics of the Ontario Research Foundation, which post he occupied until 1958. During this time and later he was frequently consulted by both industry and government on statistical matters; for example, he became a member of the Fisheries Research Board of Canada, and no doubt gave them excellent statistical advice.

In 1958 he returned to University of Toronto as Professor of Mathematics and Chairman of the Department of Mathematics, serving as Chairman until 1968. During this period he presided over an unprecedented expansion of the Department, while carefully maintaining its quality, while still maintaining his consulting work with government and industry, and while teaching. The last named should be stressed, for Dan DeLury was an immensely successful and popular lecturer, his introductory course in Probability Theory being famous throughout the university.

Professor DeLury retired as Chairman in 1968, and from the University in 1973, but continued to lecture part-time until 1977. He died after a long illness October 21, 1993.

Professor DeLury, Dan to all his friends, was noted for his generosity, both to staff and to students, his abilities as an expositor and as an administrator, and his gentle, ironic and self-deprecatory wit. He was also a noted statistician, being honoured a few years ago by the Canadian Statistical Society as one of the founders of statistics in Canada, and being the author of some twenty papers in research journals and the supervisor of three Ph.D. students. He was a great servant of the university and the mathematical and statistical community.

– Tim Rooney

The Malcolm Slingsby Robertson Fellowships and The Malcolm Slingsby Robertson Prize

Malcolm Slingsby Robertson is a distinguished research mathematician in the field of complex analysis, who completed his B.A. (1.1; i.e. first with first class honours) and M.A. at Toronto in 1929 and 1930, and his Ph.D. at Princeton in 1934. After post-doctoral research and teaching positions at Chicago and Yale he was appointed to Rutgers University in 1937, leaving as Professor Emeritus in 1966 to become the first UNIDEL Professor of Mathematics at the University of Delaware. He now lives with his wife in Kingston.

In his honour, and in recognition of his enduring commitment to research and teaching in Mathematics, the family of Professor Robertson has established an open endowment in the Department of Mathematics, University of Toronto to support The Malcolm Slingsby Robertson Fellowships for graduate students, and The Malcolm Slingsby Robertson Prize in Mathematics, to be awarded annually to a graduating Ph.D. student for demonstrated excellence in research.

The William Lowell Putnam Mathematical Competition

When December approaches, most undergraduates are getting ready for final exams. However, it is also that time of the year when many math students prepare for the "Putnam."

The William Lowell Putnam Mathematical Competition is an annual mathematical competition open to all undergraduates in colleges and universities in the United States and Canada. The Putnam is written on the first Saturday of December. It consists of two three-hour sessions, each involving six challenging problems. In 1994, 2314 contestants from 409 institutions participated in the competition.

At the University of Toronto alone, thirty-nine students participated in last year's Putnam. The competition not only provides our students an opportunity to work on interesting problems, but also helps them get to know each other and discuss their interests in mathematics, over pizza and pop provided by the Department.

The standing of a participating university depends on the performance of a designated three member team. Since 1991, the University of Toronto has consistently ranked among the top ten universities in North America. 1994 was no exception, as the team of **J.P. Grossman** (Third Year), **Naoki Sato** (Second Year), and **Edward Leung** (First Year) placed seventh. For the second year in a row, J.P. was named a Putnam Fellow for ranking among the top five individuals in North America! Congratulations to all members of the team!

Donald Coxeter Wins Joint CRM/Fields Prize

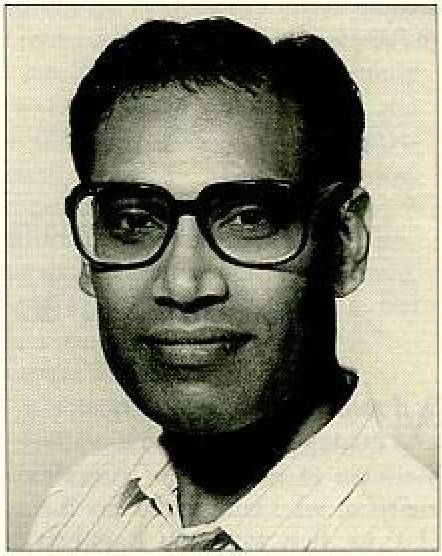
The first Joint Centre de Recherches Mathématiques/ Fields Institute Prize has been awarded to Professor **H.S.M. (Donald) Coxeter** of our Department. Prof. Coxeter was honoured for a long and remarkable record of accomplishment.

From a citation in the *Notices of the American Math. Society:* Although he has drawn inspiration from elementary geometry and the symmetries of Platonic solids, Prof. Coxeter's work has permeated modern mathematics. He has worked in a range of areas, from groups acting on *n*-space and sphere packings in *n*-dimensions, to the structure and classification of Lie groups, to noneuclidean geometry. In addition to mathematicians, many others — including artists, architects, chemists, philosophers, and physicists — know of Coxeter and have been directly influenced by his writing and his unfailing sense of beauty in mathematics. His book *Regular Polytopes* has been a classic since the time it was written and has been and continues to be widely read. The recent advances in buckyballs, fullerenes and quasicrystals have reemphasized that the regular and semiregular polytopes continue to play important roles in science and mathematics.

As part of the celebration of his Joint CRM/Fields Institute Prize, Prof. Coxeter will present lectures at the Fields Institute and the Centre de Recherches Mathématiques during the fall of 1995.

Kumar Murty Wins Steacie Fellowship

In 1995, scientists at the University of Toronto won three of the four E.W.R. Steacie Memorial Fellowships, awarded each year by the Natural Sciences and Engineering Research Council of Canada. The winners included Prof. **Kumar Murty** of our Department, cited by NSERC as "a brilliant young mathematician who is making a major contribution to number theory and arithmetic geometry, a central, but very difficult field of modern mathematics. His work explores how number relationships (such as those found, for example, in many integer equations and in the distribution of prime and non-prime numbers) are intimately tied to deep problems in geometry."



International Congress of Mathematicians, Zurich, August 1994

Two members of our Department — Profs. **George Elliott** and **John Friedlander** — were honoured by invitations to address the International Congress in Zurich. The congress meets in a different city every four years, bringing together mathematicians of all disciplines to hear the most exciting recent developments and problems in mathematics. George Elliott (who holds positions at Toronto and the University of Copenhagen) spoke in the Section on Operator Algebras and Functional Analysis, and John Friedlander in the Section on Number Theory. We are very pleased that they have summarized their addresses for our Newsletter.

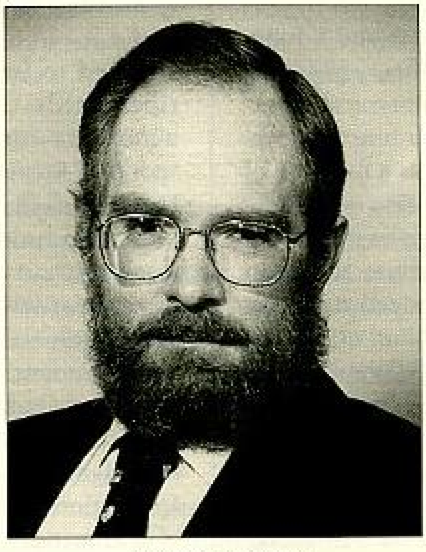
The Classification Problem for C^*-Algebras

by George A. Elliott

Ever since von Neumann showed that Heisenberg's matrix mechanics and Schrödinger's wave mechanics were equivalent formulations of the same thing (what is now called quantum mechanics) by constructing an isomorphism between the mathematical systems underlying these two epoch-making theories, the notion of isomorphism of mathematical systems has been considered of central importance, both in physics and in mathematics.

In physics, the passage from a finite to an infinite number of degrees of freedom led to a breakdown of von Neumann's uniqueness theorem. This culminated in the mathematical theory of so-called amenable von Neumann algebras, a complete enumeration of which, up to isomorphism, was given by Connes (with important contributions made by others). This enumeration, or classification, was in terms of a natural parametrization by a much simpler object — not a real number, but an action of the real numbers on a measure space.

Another recent achievement (also based on the work of a number of people) has been the enumeration, up to isomorphism, of finite simple groups. Interestingly enough, these results, both of fundamental importance for mathematical reasons alone, have been closely intertwined with the physics of infinitely many degrees of freedom — in technical terms, quantum field theory. What is even more striking, all three of these subjects have been intertwined (figuratively!) with the theory of ordinary knots, by work of Vaughan Jones.



Prof. George Elliott

I have been interested in a question which is closely related to the classification of amenable von Neumann algebras. Von Neumann algebras are self-adjoint algebras of operators on a Hilbert space which are closed in the so-called strong operator topology. It is also of interest to consider algebras which are closed in a much stronger topology, the operator norm topology. These are called C^*-algebras. While, a priori, this is a larger class, it is natural to impose a separability restriction (different in the two cases), and then the only algebras belonging to both classes are the finite-dimensional ones.

It has been known for some time that there is also a natural notion of amenability for C^*-algebras, closely related to that for von Neumann algebras. Roughly speaking, the amenable algebras, in either class, are just the ones that arise naturally.

In spite of the complete success of the work on von Neumann algebras, it was not at all clear that anything like it might be possible for C^*-algebras.

Early work on a very special case (due to Glimm, Dixmier, Bratteli, and me) had resulted in a classification, but in a setting so special as to be considered not typical at all.

After fifteen years, it finally became clear that there were too many hints. All that one had to do, it turned out, to generalize the earlier result, was to work harder. For five years, I have been doing this. Other people have been doing this, too. Soon, the problem may be solved completely — perhaps at a single stroke. The answer — what the parameters, or invariants, should be — was already there. In the special case of so-called approximately finite-dimensional C^*-algebras, solved twenty years ago, I had appropriated the tool of K-theory from algebraic topology. (This may perhaps be compared with the use of ergodic theory to parametrize amenable von Neumann algebras.) A similar use of K-theory has now led to the enumeration, up to isomorphism, a rather large class of amenable C^*-algebras. (Again, the results are due to a number of people.) The invariants now known, simple though they are, may well form a complete set.

The theory of amenable C^*-algebras, like that of amenable von Neumann algebras, appears to be related to physics. In particular, the topological nature of the invariants has been associated with the discreteness of quantum numbers. An interesting case of this is the quantum Hall effect, in which a certain quantity appears in discrete amounts to such a high degree of precision as to provide a new standard for electrical resistance. Another application has been to produce theoretical examples of quantum statistical systems with specified phase diagrams.

Bounds for *L*-Functions

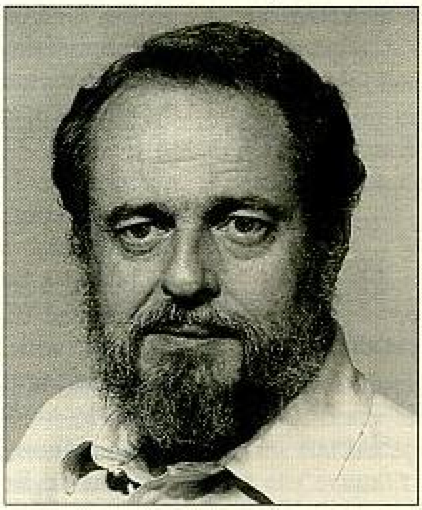
by John B. Friedlander

The study of the zeta and *L*-functions of number theory possess a rich history. The prototypical example, the "Riemann zeta-function" was considered by Euler as a function of a real variable. Euler discovered several of its basic properties, for example its special values at the even integers, and recognized its pertinence to the distribution of primes.

Riemann introduced the notion of the zeta-function as a function of a complex variable and was thereby led much further. Among other things, he proved a basic functional equation (discovered in special cases by Euler) that provides a simple relation between the values of the function at "s" and at "$1 - s$", and he studied the distribution of the zeros of the function, relating these to the error term in the Prime Number Theorem. He formulated the conjecture that the zeros (apart from those that were known to be located at the negative even integers) were all on the "critical line" $Re\ s = \frac{1}{2}$. This conjecture, known today as the Riemann Hypothesis, is among the most famous and fundamental open questions in mathematics.

Even before Riemann, other functions of similar nature had begun to appear on the scene. Dirichlet, motivated by the work of Euler, introduced his *L*-functions which played the analogous role for the distribution of primes in an arithmetic progression. Like Euler, Dirichlet considered these as functions of a real variable but, following the work of Riemann, they were analogously extended, were found to satisfy similar functional equations, and appeared also to satisfy (although no proof was forthcoming) the corresponding "Generalized" Riemann Hypothesis.

In this century the number of types of *L*-functions has proliferated and their study pervades the field. There are *L*-functions attached to number fields, *L*-functions attached to function fields (for which the corresponding Riemann Hypothesis is known to hold!), to elliptic curves, to modular forms, to motives, and to representations. Their names are associated to Dedekind, to Artin, to Hecke, to Hasse, to Weil, to Selberg, to Deligne, to Langlands. They have applications to many basic questions of arithmetic.



Prof. John Friedlander

A natural problem that has attracted attention, and is also important for applications, is that of obtaining good upper bounds for the order of magnitude of these functions.

For rather general *L*-functions, one has a functional equation of the type referred to above. This, together with the convexity principle of Phragmén-Lindelöf, allows one, without losing anything essential in generality, to restrict attention to the central critical line $Re\ s = \frac{1}{2}$. It also allows one to deduce a fairly weak but non-trivial "convexity bound". A very much stronger bound, known as the Lindelöf Hypothesis, is expected to hold, and indeed is known to follow as a simple consequence of the Generalized Riemann Hypothesis. This is the primary goal in this problem, but, for a number of applications, it is not really necessary to have such a strong bound but is crucial to have one which improves the convexity estimate.

In the case of the Riemann zeta function, ideas developed by Weyl, Hardy, Littlewood, van der Corput and others early in this century led to improvements in the convexity bound. These may be generalized to a great extent to other such functions in studying the "s-aspect" but not to the (usually more important) problem of studying the other parameters in which these more general functions vary. For example, in the case of the Dirichlet *L*-functions, there is associated the modulus "q" of the corresponding character and the dependence of the bounds on this parameter is of greater interest for applications.

In the case of the Dirichlet *L*-functions, some fifty years later (and some thirty years ago), Burgess succeeded in breaking the convexity barrier with respect to the modulus. Although extremely important, this method too is rather special. It depends crucially on the fact that, when you translate the set of integers in an interval by a small integer, you get something not very different from what you had at the start.

In my lecture I discussed some of the history and applications of these results and went on to describe a new method that I have been developing jointly with W. Duke and H. Iwaniec which improves the convexity bound in the above cases but also applies much more generally, for example to the $SL(2)$ automorphic L-functions, hence to many (conjecturally all) L-functions of elliptic curves, and to the L-functions attached to the class group of imaginary quadratic number fields.

I also discussed the related problem of finding useful approximations to L-functions by functions of simpler type, and especially by Dirichlet polynomials (by which are meant finite sections of Dirichlet series). Some basic examples of such approximations have been known for many years and furnish an important tool in the problem of obtaining bounds of the type referred to above. It would be significant, for this purpose, to find very short Dirichlet polynomials which closely approximate the L-function. I discussed recent joint work with E. Bombieri which sets limits to the length of such approximations. These limits are very close to the lengths attained by approximations already known.

Blyth Lectures in Mathematics

This distinguished lecture series was inaugurated in 1993 as part of the Department's 150th Anniversary Celebrations.

The 1994 Blyth lecturer was Prof. **Charles Fefferman** (Princeton), an analyst and mathematical physicist who won the Fields Medal in 1978. Fefferman gave a series of talks on mathematical problems that appear in quantum mechanics. He explained why the laws of quantum statistical mechanics imply the formation of atoms, molecules and matter as we know them. He presented a statistical analysis of the ground state structure of the atom, in the limit when the nuclear charge is large. In a final talk he explained that the stability of matter amounts to a simple mathematical inequality that has yet to be rigorously established, but which may be the key to understanding many physical problems.

Fefferman's lectures were based in part on his joint work with Prof. Luis Seco of our Department, that was cited in Encyclopedia Britannica's 1995 Yearbook.

This year's Blyth lecturer was the 1994 Fields Medalist, Prof. **Efim Zelmanov** (Chicago, Yale), an algebraist who gave three lectures on the Burnside Problem and Profinite Groups. W. Burnside formulated his famous problem, "Is a finitely generated periodic group finite?" in 1902. In the following 90 + years the problem played an outstanding role in the development of algebra. Zelmanov reviewed the history and recent developments connected with the problem — it was his solution of the "restricted Burnside problem" that gained him the Fields Medal. He elucidated the importance of the Burnside problems to group theory and Lie algebras.

On Fermat's Last Theorem

by V. Kumar Murty

Fermat's Last Theorem is the assertion that if $n \geq 3$ is an integer, and x, y, z are integers satisfying the equation

$$x^n + y^n = z^n$$

then $xyz = 0$ (i.e., at least one of x, y or z is zero). Pierre de Fermat was a lawyer and amateur mathematician. He made many mathematical assertions without supplying proofs. His "Last Theorem" is the last of his claims that had not been already either proved rigorously or disproved.

Many mathematicians since the time of Fermat have studied his equation. Most met with little or no success. There were several exceptions, most notably Kummer. In the last ten years, it was the work of Frey relating Fermat's equation to elliptic curves which led to rapid progress. What is an elliptic curve? Over the field of rational numbers Q, it is a curve given by an equation of the form

$$y^2 = x^3 + ax + b$$

where a, b are rational numbers and the cubic on the right has distinct roots. It should not be confused with an ellipse! Elliptic curves have the marvellous property that the set of points forms an additive group.

Assuming that (a,b,c) is a nontrivial solution of Fermat's equation and that $n = p$ is prime, Frey considered the elliptic curve

$$y^2 = x(x-a^p)(x+b^p)$$

and showed that the points of order p on this curve generate an extension $K(p)$ of Q which has very limited ramification.

From another direction, in the 1950's, Taniyama initiated a line of thought which was later developed by Shimura and Weil, and which led to the conjecture that all elliptic curves defined over Q could be classified in terms of modular cusp forms. Cusp forms are analytic objects but when coupled with an algebra of operators called the Hecke algebra, they yield arithmetic information. They come equipped with several parameters, one of which is the "level".

Serre had the insight to consider cusp forms which are "congruent" to a given elliptic curve modulo the prime p. He predicted that the ramification of the field $K(p)$ is related to the level of such cusp forms. In particular, the very limited ramification of $K(p)$ should imply the existence of a cusp form of level 2. But such forms are known not to exist!

Elliptic curves which are constructed from modular (cusp) forms are called modular elliptic curves. Ribet was able to prove that Serre's insight was correct for such elliptic curves. Thus, if the Frey curve is indeed modular, as the Shimura-Taniyama conjecture asserts, then we get a contradiction and Fermat's Last Theorem follows.

The final thrust was given by Andrew Wiles who showed that semistable elliptic curves over Q (a class of curves which includes Frey's) are indeed modular. He uses the techniques of deformation theory of Galois representations initiated a few years earlier by Mazur, and an intense study of Hecke algebras, building on work of Hida, Mazur and many others. A key property of these algebras was established by Wiles in collaboration with Richard Taylor.

Wiles' work has now appeared in the Annals of Mathematics. Number theorists around the world are studying it and trying to explain it to each other and to the world. The Canadian Mathematical Society and the American Mathematical Society will soon be publishing a volume with some of the background necessary to understand the argument. Wiles' work is a remarkable achievement for the number and variety of its new insights.

Recent Faculty Appointments

Claudio Albanese joined the Department at Erindale College in 1994. Prof. Albanese was born in Ortona, Italy. He did his undergraduate work in Physics at the University of Pavia, and obtained his Ph.D. in Mathematical Physics from the Swiss Federal Institute of Technology in Zurich. He spent the following three years in the U.S., at U.C.L.A., the Courant Institute and Princeton University, and then returned to Zurich for another three years.

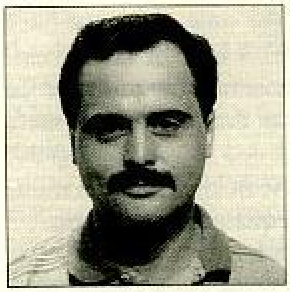
Prof. Albanese has worked on infinite-dimensional Hamiltonian systems, proving the first bifurcation theorem for a system with dense spectrum, and in statistical mechanics, where he introduced powerful new techniques for the analysis of quantum spin systems. He is currently interested in problems of stochastic control theory that have applications to mathematical finance.

Boris Khesin has accepted a position in our Department this year. Prof. Khesin was born in Moscow and received his Ph.D. in 1989 from Moscow State University, where he was a student of Prof. V.I. Arnold. He left Russia the following year. After spending two years at the University of California in Berkeley, he held an Assistant Professorship at Yale. He has been a Visiting Professor at the Max-Planck Institute in Bonn, the Institut des Hautes Etudes Scientifiques in Bures-sur-Yvette, and the Isaac Newton Institute in Cambridge.

Prof. Khesin has made important contributions to symplectic geometry, singularity theory and integrable systems, all with a highly individual flavour. His idea of the logarithm of the derivative (now called the Khesin cocycle) has provided a missing link in the theory of integrable systems and determinant theory. He has developed this idea in his recent work on group-theoretical methods in hydrodynamics, providing a beautiful generalization to infinite-dimensional mechanics of the notion of ergodic behaviour.

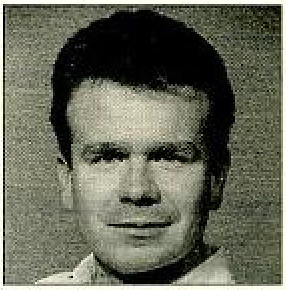
Askold Khovanskii has also joined our Department this year, as Professor. One of the leading Russian mathematicians of his generation, Prof. Khovanskii has been a member of the Institute for System Studies of the Academy of Sciences of the U.S.S.R., and Scientific Director of the Moscow Mathematical Institute. He has held visiting professorships in Canada, France, Sweden, Switzerland and the United States, and was invited to address the International Congress of Mathematicians in Warsaw in 1983.

Prof. Khovanskii is a geometer, but his work transcends classification into a fixed subject area. His theory of "fewnomials" — the idea that objects defined by "simple" systems of equations should have a "simple" topology — has important applications in real algebraic geometry, the theory of abelian integrals, logic and complexity theory. His foundational work on Newton polyhedra connects real and complex algebraic geometry with the geometry and combinatorics of convex polyhedra, with stunning applications in both directions.



Alexander Nabutovsky joined the Department of Mathematics at Toronto in 1993. He began his studies in Russia, and obtained his Ph.D. from the Weizmann Institute of Science in Israel, in 1992. He has taught at Stanford University and the Courant Institute of Mathematical Sciences at New York University.

Prof. Nabutovsky is interested in questions of existence and complexity of algorithms for problems in differential geometry and topology. Using recursion theory, he has shown that the theory of "thick" knots differs in a remarkable way from classical knot theory (where the "ropes" have zero thickness!). He is using algorithmic methods to study one of the most important questions in Riemannian geometry: What is the "best" metric on a given compact manifold?



Maciej Zworski is taking a position as Professor in our Department this year. He was born in Wroclaw, Poland, and began his studies in Mathematics at Imperial College, London. When his family moved to Canada in 1983, he continued his studies at the Massachusetts Institute of Technology. He received his Ph.D. from M.I.T. in 1989, as a Sloan Doctoral Dissertation Fellow. After graduating, Prof. Zworski was a Benjamin Pierce Lecturer at Harvard, and a Visiting Professor at the Université de Paris-Sud in Orsay. He went to Johns Hopkins University in Baltimore as a Sloan Research Fellow in 1991, and became Professor at Johns Hopkins in 1994.

Prof. Zworski is a world expert in partial differential equations, microlocal analysis and scattering theory. He has done remarkable work on the diffraction of waves by solid bodies, and on the distribution of resonances or poles in obstacle and potential scattering. Prof. Zworski joins a very strong group in partial differential equations and mathematical physics at Toronto.

Recent limited-term faculty appointments: **François Destrempes** (Ph.D., Cornell) was formerly Assistant Professor at the University of Ottawa. **Eriko Hironaka** (Ph.D., Brown); Prof. Hironaka was a Szegö Instructor at Stanford before coming to Toronto. **Andrew Hwang** (Ph.D., Berkeley) was a Fellow at Osaka University before joining our Department. **Lawrence Kolasa** (Ph.D., California Inst. Technology), formerly at the University of Wisconsin-Parkside. **Serge Resnick** (Ph.D., Chicago). **Brooks Roberts** (Ph.D., Chicago); before coming to Toronto, Prof. Roberts was Guest Researcher at the Max-Planck-Institut für Mathematik. **Frank Sottile** (Ph.D., Chicago). **Sarah Witherspoon** (Ph.D., Chicago).

Prof. Warwick Sawyer Writes From Cambridge, England

I was very pleased to see that the Department is taking an active part in regard to mathematics teaching in elementary schools. Until 1976 I held a joint appointment, Mathematics and Education, and have always regarded the elementary stage as the vital one. If a child comes to dislike or fear mathematics at that stage it requires a near miracle to correct these feelings later.

In all the countries I have been in, I have found that those people who have never experienced good mathematics teaching: 1. Think doing mathematics is a matter of obeying certain rules (how these rules came to be revealed to humanity is a question that is never raised). 2. Thus it is an extremely dull subject. 3. It is difficult. 4. Often it is found frightening. Nobody can possibly teach mathematics well so long as he or she is in the grip of this culture. An approach that sets out simply to convey information is bound to fail. The emotional and intellectual basis must be tackled from the start.

At the time of "Modern Math" in U.S.A. many lecturers emphasized how advanced the new topics were. "Do you realize that this topic was never taught at a lower level than college?" This was exactly the opposite of what was needed, something so simple that it produced the response, "Oh, I see that. Why did nobody tell me this before?"

When I was giving talks to elementary school teachers in Connecticut I managed to get this reaction by means of pictures which showed that the sum of even numbers was always even, and so forth. As most of the teachers had never *seen* a mathematical result before, the effect was almost that of a religious revival meeting.

Teachers and future teachers should get the experience of arriving at mathematical results for themselves; they should meet results that are interesting and surprising; most of the time they should succeed and gain in confidence; they should also gradually come to realize that some problems in mathematics have taken years or even centuries to solve, and that some are still unsolved. It should be emphasized that in our trade we we do not claim to know everything. There are questions, like Goldbach's conjecture, that can be understood by an eight-year-old student, to which nobody knows the answer.

Alumni News

John Benedetto (Ph.D. '64) is a professor of mathematics at the University of Maryland at College Park. He is the founder and editor of the Journal of Fourier Analysis and Applications.

Jean Duquesnay retired in 1984 after teaching secondary school math for 28 years. She has been a climatological weather observer since 1957 and is Membership Secretary of the Algoma Unit of Superannuated Teachers of Ontario.

Samuel Goldberg (Ph.D. '51) has been a visiting professor at the University of the West Indies (Kingston, Jamaica) since 1991.

Ken Hoyle (4T3) and his wife Donnie have lived in Guelph, Ontario since Ken's retirement in 1981 from Bell Canada. He worked for Bell for 35 years, and travelled to many places (Geneva, Stockholm, Moscow, Australia, etc.) for international telephony. "It was not at all boring as Sam Beatty foretold in '45". He was a member of the winning Putnam team in '42 and recalls that, after the morning session, Donald Coxeter "provided off-the-cuff answers for those [problems] I had found puzzling".

Bruce MacDonald writes: "I suspect actuarial science is no longer as important at Varsity as it once was, what with the courses being given at Waterloo and Laval that are specifically designed to produce actuaries. We didn't go into first year planning to become actuaries. It was a decision we reached *en route* to our degree, and I think we were the better for this."

Rocco Martino (5T1, Ph.D. '55 Inst. Aerospace Studies) has held academic appointments at the University of Waterloo, where he was the founding head of the Dept. of Management Engineering, and at New York University. He is the founder and Chairman of XRT, Inc. (Wayne, Pennsylvania) which provides real-time financial networks on a national and international basis. He has published 18 books and over 200 articles. Neumann College (Aston, Pa.) awarded Rocco an honorary D.Sc. in 1993, citing him and his wife Barbara as "an example for compassion and commitment to the improvement of the larger world around them".

Clare Stockdale (6T7) has recently returned to her home in Sydney, Australia after working as a U.S. tax associate with Price-Waterhouse in Tokyo, where her husband was on assignment as a corporate lawyer. She writes "I would love to hear about fellow graduates from MPC. I only ran into 3 of them at the 25th Reunion. I believe there were only 10 girls who graduated in that class but our attrition rate was less than 40% whereas the boys' was over 75%. Even so, I wonder how many of the girls actually ended up with full-time careers."

Robert Thomas (6T4) graduated from M & P with philosophy replacing the more usual physics. He has worked in the Dept. of Applied Math., University of Manitoba, since 1978, and edits the new Series III of Philosophia Mathematica, the only journal in the world dedicated to the philosophy of mathematics. His son, Hugh (9T4) is now a graduate student in Mathematics at the University of Chicago.

George Watson (3T6) has been a fellow of the Society of Actuaries for 50 years and of the Canadian Institute of Actuaries since its incorporation 30 years ago.

Thinking About Making a Donation?

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