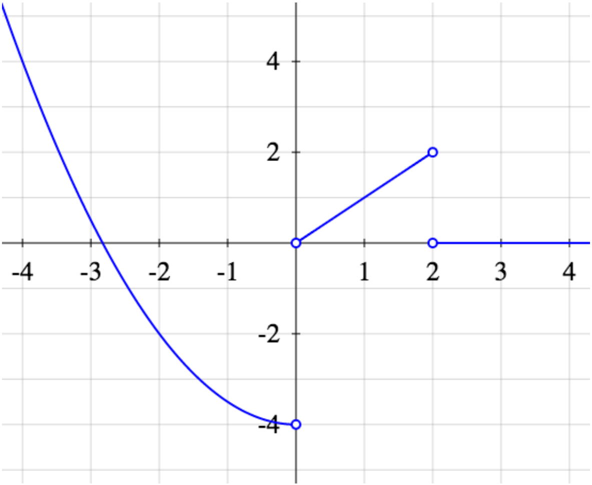
Announcements

- **Topics:** Proof of differentiation rules, chain rule, trig differentiation and implicit differentiation
- **Quiz 1** today.
- **Homework:** Watch videos 4.1 - 4.11

Intuitive idea of the derivative

Below is the graph of the derivative of some function f. We know f is continuous and $f(0) = 0$. Graph f.



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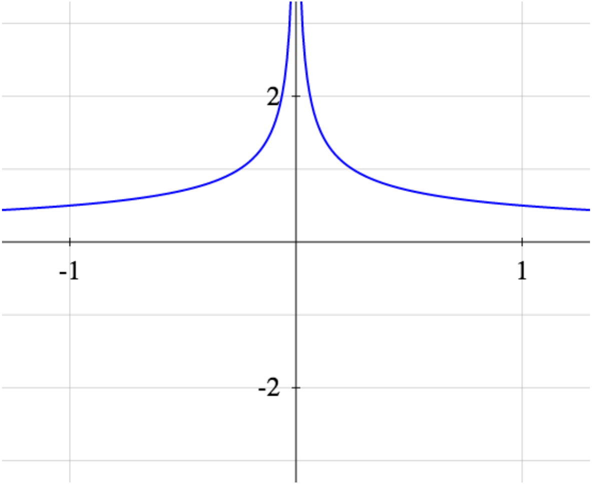


Derivatives from the definition

Let
$$g(x) = \frac{2}{\sqrt{x}}$$
Calculate $g'(4)$ directly from the definition of derivative as a limit.

Estimations

Without using a calculator, estimate $\sqrt[20]{1.01}$ as well as you can.

Hint: The tangent line is a good approximation to the function near the point of tangency.

Higher order derivatives

Let $g(x) = \dfrac{1}{x^3}$.

Calculate the first few derivatives.

Make a conjecture for a formula for the n-th derivative $g^{(n)}(x)$. Homework: Prove it by induction.

Homework: Computations

Compute the derivative of the following functions:

1. $$f(x) = x^{100} + 3x^{30} - 2x^{15}$$
2. $f(x) = \sqrt[3]{x} + 6$
3. $f(x) = \dfrac{4}{x^4}$
4. $f(x) = \sqrt{x}\,(1 + 2x)$
5. $f(x) = \dfrac{x^6 + 1}{x^3}$
6. $f(x) = \dfrac{x^2 - 2}{x^2 + 2}$

Quotient rule

Let $a \in \mathbb{R}$.

Given functions f and g defined in a neighbourhood of a.

Define $h(x) = \frac{f(x)}{g(x)}$.

Assume f and g are _______________.

Assume _________________________.

Then ____.

Prove this.

True or False - Differentiability and Composition

Let f and g be functions with domain $\mathbb{R}$. Let $c \in \mathbb{R}$.
Assume f and g are differentiable at c.
What can we conclude?

1. $f \circ g$ is differentiable at c.
2. $f \circ f$ is differentiable at c.
3. $f \circ \sin$ is differentiable at c.
4. $\sin \circ f$ is differentiable at c.

Quick composition

Let f and g be differentiable functions and let $h = f \circ g$. What is $h'(2)$?

1. $f'(2) \circ g'(2)$
2. $f'(2)g'(2)$
3. $f'(g(2))g'(2)$
4. $f'(g(x))g'(2)$

Warm up

Differentiate

$$f(x) = \sqrt{x + \sqrt{x+1}}.$$

A long chain

The function below has 137 square roots:

$$f(x) = \sqrt{x + \sqrt{x + \sqrt{x + \sqrt{x + \ldots + \sqrt{x + \sqrt{x + 1}}}}}}$$

Find the equation of the line tangent to the graph of f at the point with x-coordinate 0.

Derivative of cos

Let $g(x) = \cos x$.

Obtain and prove a formula for its derivative directly from the definition of derivative as a limit.

Hint: Imitate the derivation in Video 3.12.
You will need a trig identity. Google it if you do not know it.

Homework: Derivatives of the other trig functions

Use the basic differentiation rules, as well as

$$\frac{d}{dx}\sin x = \cos x, \qquad \frac{d}{dx}\cos x = -\sin x,$$

to quickly obtain and prove formulas for the derivatives of tan, cot, sec, and csc.

Homework: Trig derivatives

Compute the derivatives of the following functions:

1. $f(x) = \tan(3x^2 + 1)$
2. $f(x) = (\cos x)(\sin 2x)(\tan 3x)$
3. $f(x) = \cos(\sin(\tan x))$
4. $f(x) = \cos\left(3x + \sqrt{1 + \sin^2 x^2}\right)$

A pesky function

Let $h(x) = \begin{cases} x^2 \sin \dfrac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}.$

1. Calculate $h'(x)$ for any $x \neq 0$.
2. Using the definition of derivative, calculate $h'(0)$.
3. Calculate $\lim\limits_{x \to 0} h'(x)$

   Hint: Questions 2 and 3 have different answers.
4. Is h continuous at 0?
5. Is h differentiable at 0?
6. Is h' continuous at 0?

Differentiable functions

Let $a \in \mathbb{R}$.
Let f be a function with domain $\mathbb{R}$.
Assume f is differentiable everywhere.
What can we conclude?

1. $f(a)$ is defined.
2. $\lim_{x \to a} f(x)$ exists.
3. f is continuous at a.
4. $f'(a)$ exists.
5. $\lim_{x \to a} f'(x)$ exists.
6. f' is continuous at a.

Implicit differentiation

The equation

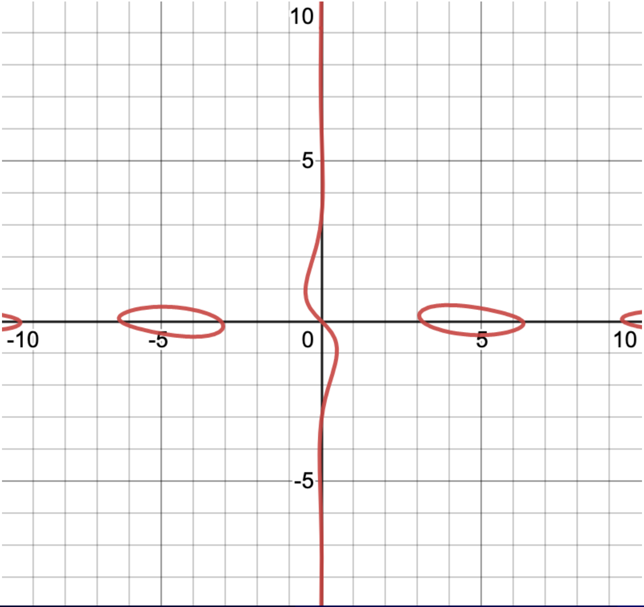
$$\sin(x + y) + xy^2 = 0$$

defines a function $y = h(x)$ near $(0, 0)$. graph

Using implicit differentiation, compute

1. $h(0)$
2. $h'(0)$
3. $h''(0)$
4. $h'''(0)$

Graph



Quiz info

1. Quiz 1 is available on the Quizzes section of the MAT137 Quercus site.
2. If you have any question or technical issue, contact me **privately on the Zoom chat**.
3. You **cannot post anything related to the Quiz anywhere until Saturday, this includes the group chat, Piazza and anywhere else**. There may very well be students who are still writing the quiz after you finish, **even after noon**.
4. Commented solutions to the quiz questions will be posted next week and special office hours will be held to discuss the quiz questions.