Announcements

- **Topics:** IVT, EVT, Derivatives
- **Thematic office hours** are available! See office hour schedule for more info.
- **Quiz 1** will be on Friday and take 30 minutes. It will cover up to and until video 2.20. You will be able to access it from 11:20am - noon. You must start the quiz before 11:30am to get full time.
- **Homework:** Watch videos 3.6, 3.7, 3.9 - 3.13

Trig computations

Using that $\lim_{x\to 0} \frac{\sin x}{x} = 1$, compute the following limits:

1. $\lim_{x\to 2} \frac{\sin x}{x}$
2. $\lim_{x\to 0} \frac{\sin(5x)}{x}$
3. $\lim_{x\to 0} \frac{\tan^2(2x^2)}{x^4}$
4. $\lim_{x\to 0} \frac{\sin e^x}{e^x}$
5. $\lim_{x\to 0} \frac{1-\cos x}{x}$
6. $\lim_{x\to 0} \frac{\tan^{10}(2x^{20})}{\sin^{200}(3x)}$
7. $\lim_{x\to 0} [(\sin x)(\cos(2x))(\tan(3x))(\sec(4x))(\csc(5x))(\cot(6x))]$

Compute:

1. $\lim_{x\to\infty} \left(x^7 - 2x^5 + 11\right)$
2. $\lim_{x\to\infty} \left(x^2 - \sqrt{x^5 + 1}\right)$
3. $\lim_{x\to\infty} \frac{x^2 + 11}{x + 1}$
4. $\lim_{x\to\infty} \frac{x^2 + 2x + 3}{3x^2 + 4x + 5}$
5. $\lim_{x\to\infty} \frac{x^3 + \sqrt{2x^6 + 1}}{2x^3 + \sqrt{x^5 + 1}}$

Plus or minus infinity?

Compute:

1. $$\lim_{x \to -3^+} \frac{x^2 - 9}{3 - 2x - x^2}$$
2. $$\lim_{x \to 1^+} \frac{x^2 - 9}{3 - 2x - x^2}$$

Is this computation correct?

Compute $L = \lim_{x \to -\infty} \left[x - \sqrt{x^2 + x} \right]$.

Solution 1

$$L = \lim_{x \to -\infty} \frac{\left[x - \sqrt{x^2 + x} \right] \left[x + \sqrt{x^2 + x} \right]}{\left[x + \sqrt{x^2 + x} \right]} = \lim_{x \to -\infty} \frac{x^2 - (x^2 + x)}{\left[x + \sqrt{x^2 + x} \right]}$$

$$= \lim_{x \to -\infty} \frac{-x}{x \left[1 + \sqrt{1 + \frac{1}{x}} \right]} = \lim_{x \to -\infty} \frac{-1}{\left[1 + \sqrt{1 + \frac{1}{x}} \right]} = \frac{-1}{2}$$

Temperature

Yesterday, the outside temperature in Toronto at 6 AM was 15°. At 6 PM, the temperature was 30°.

1. Must there have been a time between 6 AM and 6 PM when the temperature was 20°? Explain how you know. Which assumption about temperature allows you to reach your conclusion?
2. Must there have been a time between 6 AM and 6 PM when the temperature was 32°? Explain how you know.
3. Could there have been a time between 6 AM and 6 PM when the temperature was 32°? Explain how you know.

Existence of solutions

Prove that the equation

$$x^4 - 2x = 100$$

has at least two solutions.

Definition of maximum

Let f be a function with domain I.
Which one (or ones) of the following is (or are) a definition of "f has a maximum on I"?

1. $\forall x \in I, \exists C \in \mathbb{R}$ s.t. $f(x) \leq C$
2. $\exists C \in \mathbb{R}$ s.t. $\forall x \in I, f(x) \leq C$
3. $\exists C \in \mathbb{R}$ s.t. $\forall x \in I, f(x) < C$

More on the definition of maximum

Let f be a function with domain I.
What does each of the following mean?

1. $\exists C \in \mathbb{R}$ s.t. $\forall x \in I$, $f(x) \leq C$
2. $\exists C \in \mathbb{R}$ s.t. $\forall x \in I$, $f(x) < C$
3. $\exists a \in I$ s.t. $\forall x \in I$, $f(x) \leq f(a)$
4. $\exists a \in I$ s.t. $\forall x \in I$, $f(x) < f(a)$

EVT is best possible?

Recall the statement of EVT.

Find/draw a continuous function f which is continuous on $[0, 1)$ which doesn't have a maximum.

Find/draw a continuous function f which is continuous on $[0, 1)$ which has neither a maximum nor a minimum.

Extrema

In each of the following cases, does the function f have a maximum and a minimum on the interval I?

1. $f(x) = x^2, \quad I = (-1, 1).$
2. $f(x) = \dfrac{(e^x + 2)\sin x}{x} - \cos x + 3, \quad I = [2, 6]$

Tangent line to a line?

What is the equation of the line tangent to the graph of $y = x$ at the point with x–coordinate 7?

1. $y = x + 7$
2. $y = x$
3. $y = 7$
4. $x = 7$
5. There is no tangent line at that point.
6. There is more than one tangent line at that point.

Absolute value and derivatives

Let $h(x) = x|x|$. What is $h'(0)$?

1. It is 0.
2. It does not exist because $|x|$ is not differentiable at 0.
3. It does not exist because the right- and left-limits, when computing the derivative, are different.
4. It does not exist because it has a corner.

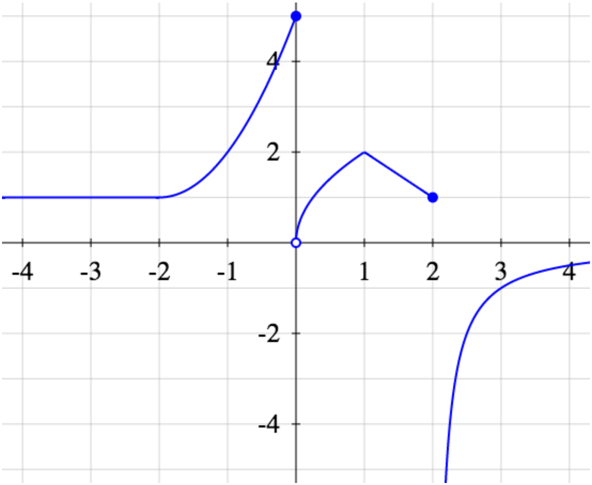
Prove these statements are false with counterexamples

Let C be a curve. Let P be a point in C.

1. The line tangent to C at P
   intersects C at only one point: P.
2. If a line intersects C only at P,
   then that line must be the tangent line to C at P.
3. The tangent line to C at P intersects C at P
   and "does not cross" C at P.
   (This means that, near P, it stays on one side of C.)
4. If a line intersects C at P
   and "does not cross" C at P,
   then it is the tangent line to C at P.

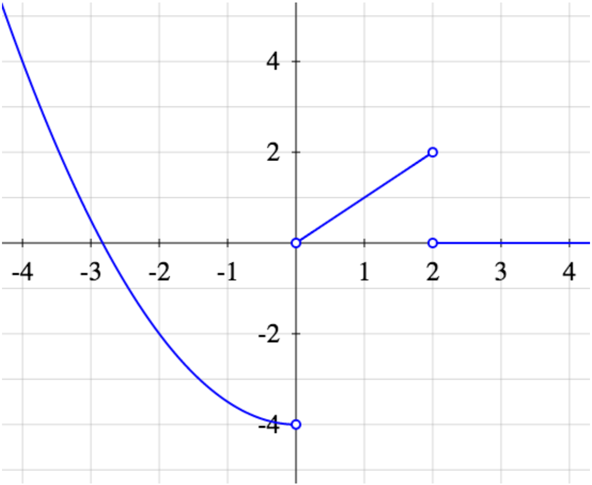
Intuitive idea of the derivative

Graph the derivative of this function.



Intuitive idea of the derivative

Below is the graph of the derivative of some function f. We know f is continuous and $f(0) = 0$. Graph f.



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